Day 23

1) $\int \sin^{-3/2}x \cos^3 x\,dx = \int \sin^{-3/2}x \cos^2 x \cos x\,dx = \int \sin^{-3/2}x\,(1-\sin^2 x)\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$= \int u^{-3/2}(1-u^2)\,du = \int u^{-3/2} - u^{1/2}\,du$

$= -2u^{-1/2} - \frac{2}{3}u^{3/2} + C = \frac{-2}{\sqrt{\sin x}} - \frac{2}{3}\sin^{3/2}(x) + C$

#2 $\int \tan^9 x \sec^4 x\,dx = \int \tan^9 x \sec^2 x \sec^2 x\,dx = \int \tan^9 x(1+\tan^2 x)\sec^2 x\,dx$

$u = \tan x,\ du = \sec^2 x\,dx$

$= \int u^9(1+u^2)\,du = \int u^9 + u^{11}\,du = \frac{u^{10}}{10} + \frac{u^{12}}{12} + C = \frac{\tan^{10}x}{10} + \frac{\tan^{12}x}{12} + C$

#3 $\int \cos^2(x/3)\,dx = \int \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2x}{3}\right)dx = \frac{1}{2}x + \frac{3}{4}\sin\left(\frac{2x}{3}\right) + C$

Now Reduce

#4 $\int \cos^4(3x)\,dx = \frac{1}{3}\int \cos^4 u\,du = \frac{1}{3}\left[\frac{1}{4}\cos^3 u \sin u + \frac{3}{4}\int \cos^2 u\,du\right]$

convert

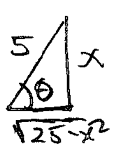
$u = 3x$
$du = 3dx$
$1/3\,du = dx$

$= \frac{1}{3}\left[\frac{1}{4}\cos^3 u \sin u + \frac{3}{4}\int \frac{1}{2} + \frac{1}{2}\cos(2u)\,du\right]$

$= \frac{1}{3}\left[\frac{1}{4}\cos^3 u \sin u + \frac{3}{8}u + \frac{3}{16}\sin(2u)\right] + C$

$= \frac{1}{12}\cos^3(3x)\sin(3x) + \frac{1}{8}(3x) + \frac{1}{16}\sin(6x) + C$

#5 $\int \frac{x^2}{\sqrt{25-x^2}}\,dx = \int \frac{25\sin^2\theta\, 5\cos\theta\,d\theta}{5\cos\theta} = 25\int \sin^2\theta\,d\theta = 25\int \frac{1}{2} - \frac{1}{2}\cos(2\theta)\,dx$



$x = 5\sin\theta$
$dx = 5\cos\theta\,d\theta$
$\sqrt{25-x^2} = 5\cos\theta$

$= 25\left[\frac{\theta}{2} - \frac{1}{4}\sin(2\theta)\right] + C$

$\sin 2\theta = 2\cos\theta\sin\theta$

$= 25\left[\theta/2 - \frac{1}{2}\sin\theta\cos\theta\right] + C$

$= 25\left[\frac{\arcsin(x/5)}{2} - \frac{1}{2}\,\frac{x}{5}\cdot\frac{\sqrt{1-x^2}}{5}\right] + C$

$= \frac{25}{2}\arcsin(x/5) - \frac{1}{2}\sqrt{1-x^2} + C$

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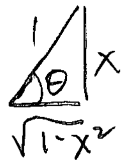
#6 $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^3\theta\cos\theta\,d\theta}{\cos\theta} = \int \sin^3\theta\,d\theta$



$x = \sin\theta$

$dx = \cos\theta\,d\theta$

$\sqrt{1-x^2} = \cos\theta$

$$= -\frac{1}{3}\sin^2\theta\cos\theta + \frac{2}{3}\int \sin\theta\,d\theta$$

$$= -\frac{1}{3}\sin^2\theta\cos\theta - \frac{2}{3}\cos\theta + C$$

use triangle $\displaystyle = -\frac{1}{3}x\sqrt{1-x^2} - \frac{2}{3}\sqrt{1-x^2} + C$

#7 $\displaystyle\int x\cos^2 x\,dx = \int x\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx$

$$= \int \frac{x}{2}\,dx + \int \frac{x}{2}\cos 2x\,dx$$

$u = \frac{x}{2}$ $\quad dv = \cos 2x\,dx$

$du = \frac{1}{2}dx$ $\quad v = \frac{1}{2}\sin 2x$

$$= \frac{x^2}{4} + \overset{uv}{\frac{x}{4}\sin 2x} \overset{-\int v\,du}{-\int \frac{1}{4}\sin 2x\,dx}$$

$$= \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + C$$