Day 32 Math 131

#1 a) $\sum_{n=0}^{\infty} (2/3)^n = \frac{a}{1-r} = \frac{1}{1-2/3} = \boxed{3}$ or $\sum_{n=0}^{\infty} (2/3)^n = 1 + 2/3 + (2/3)^2 + \cdots$ $a=1$, $r=2/3$, $|r|<1$

b) $\sum_{n=0}^{\infty} 4(-2/5)^n = 4 + 4(-2/5) + 4(-2/5)^2 + \cdots$ $a=4$, $r=-2/5$, $|r|<1$

$\sum_{n=0}^{\infty} 4(-2/5)^n = \frac{a}{1-r} = \frac{4}{1-2/5} = \frac{4}{7/5} = \boxed{20/7}$

c) $\sum_{n=0}^{\infty} 6(5/4)^n = 6 + 6(5/4) + 6\cdot(5/4)^2 + \cdots$ $a=6$, $r=5/4$,

$|r|>1$ Series $\boxed{\text{Diverges}}$

#2 a) $\sum_{k=0}^{\infty} (4/3)^{-k} = 1 + 3/4 + (3/4)^2 + \cdots$ $a=1$, $r=3/4$; $|r|<1$ $\sum_{k=0}^{\infty} (4/3)^{-k} = \frac{1}{1-3/4} = \boxed{4}$

b) $\sum_{n=0}^{\infty} 3(2/5)^{2n} = 3 + 3(2/5)^2 + 3(2/5)^4 + \cdots$ $a = 3$, $r=(2/5)^2$ $\sum_{n=0}^{\infty} 3(2/5)^{2n} = \frac{a}{1-r} = \frac{3}{1-4/25} = \frac{75}{21} = \boxed{\frac{25}{7}}$

c) $\sum_{k=1}^{\infty} 4(1/3)^k = 4\cdot 1/3 + 4\cdot(1/3)^2 + 4(1/3)^3 + \cdots = \frac{a}{1-r} = \frac{4/3}{1-1/3} = \boxed{2}$ $r=1/3$

d) $\sum_{k=2}^{\infty} 3(-1/2)^k = 3\cdot(-1/2)^2 + 3(-1/2)^3 + 3(-1/2)^4 + \cdots = \frac{a}{1-r} = \frac{3/4}{1+1/2} = \boxed{\frac{1}{2}}$ $r=-1/2$ $|r|<1$

#3 $8 + 6 + \frac{9}{2} + \frac{27}{8} + \cdots = \frac{a}{1-r} = \frac{8}{1-3/4} = \boxed{32}$ $a=8$, $r=3/4$ $|r|<1$

#4 $\sum_{k=0}^{\infty} \left(\frac{2}{k+2} - \frac{2}{k+3}\right)$

$$\begin{aligned} S_n = &\left(\tfrac{2}{2} - \tfrac{2}{3}\right) \\ &+ \left(\tfrac{2}{3} - \tfrac{2}{4}\right) \\ &+ \left(\tfrac{2}{4} - \tfrac{2}{5}\right) \\ &+ \left(\tfrac{2}{5} - \tfrac{2}{6}\right) \\ &\vdots \\ &+ \left(\tfrac{2}{n} - \tfrac{2}{n+1}\right) \\ &+ \left(\tfrac{2}{n+1} - \tfrac{2}{n+2}\right) \\ &+ \left(\tfrac{2}{n+2} - \tfrac{2}{n+3}\right) \end{aligned}$$

$S_n = \frac{2}{2} - \frac{2}{n+3}$

So $\sum_{k=0}^{\infty} \left(\frac{2}{k+2} - \frac{2}{k+3}\right) = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{2}{2} - \frac{2}{n+3} = \boxed{1}$

#5 $\sum_{k=0}^{\infty} \ln\left(\frac{k+2}{k+1}\right) = \sum_{k=0}^{\infty} \ln(k+2) - \ln(k+1)$

$$\begin{aligned} S_n = &\ln(2) - \ln 1 \\ &+ \ln(3) - \ln(2) \\ &+ \ln(4) - \ln(3) \\ &\vdots \\ &+ \ln(n) - \ln(n-1) \\ &+ \ln(n+1) - \ln(n) \\ &+ \ln(n+2) - \ln(n+1) \end{aligned}$$

$S_n = \ln(n+2) - \ln(1) = \ln(n+2)$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \ln(n+2) = \infty$

So $\sum_{k=0}^{\infty} \ln\left(\frac{k+2}{k+1}\right) = \infty$ (Diverges)

#6 $\sum_{k=1}^{\infty} \arctan(k+1) - \arctan(k)$; $S_n = \arctan(2) - \cancel{\arctan 1}$
$+ \cancel{\arctan 3} - \cancel{\arctan 2}$
$+ \arctan 4 - \cancel{\arctan 3}$
$\vdots$
$+ \arctan(n-1) - \arctan(n-2)$
$+ \arctan(n) - \arctan(n-1)$
$+ \arctan(n+1) - \cancel{\arctan(n)}$

$S_n = \arctan(n+1) - \arctan 1$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \arctan(\overset{\nearrow\infty}{n+1}) - \arctan(1)$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$$

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#10 $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$, n^{th} term test $\lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1+1/n^2} = \frac{0}{1} = 0$

The test is $\boxed{\text{inconclusive}}$

#12 $\sum_{k=1}^{\infty} \frac{k^2}{2^k}$; n^{th} term test $\lim_{n\to\infty} \frac{n^2 \nearrow\infty}{2^n \searrow\infty} = \lim_{x\to\infty} \frac{x^2}{2^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{\ln 2 \cdot 2^x}$

$$= \lim_{x\to\infty} \frac{2}{(\ln 2)^2 \cdot 2^x} = 0$$

Test is $\boxed{\text{inconclusive}}$

#14 $\sum_{k=1}^{\infty} \frac{k^3}{k^3+1}$; n^{th} term test; $\lim_{n\to\infty} \frac{n^3}{n^3+1} = \lim_{n\to\infty} \frac{1}{1+1/n^3} = 1 \quad (\neq 0)$

The series $\boxed{\text{Diverges}}$