Day 35 Math 131

#1 p648 #20 $\sum_{k=1}^{\infty} \left(\frac{2k}{k+1}\right)^k$. The terms are positive. Apply the root test.

$$r = \lim_{k\to\infty} \sqrt[k]{\left(\frac{2k}{k+1}\right)^k} = \lim_{k\to\infty} \frac{2k}{k+1} = \lim_{k\to\infty} \frac{2}{1+1/k} = 2 > 1$$

By the root test since $r > 1$, the series diverges

#22 $\sum_{k=1}^{\infty} \left(1+\frac{3}{k}\right)^{k^2}$. Terms are positive. Apply root test

$$r = \lim_{k\to\infty} \sqrt[k]{(1+3/k)^{k^2}} = \lim_{k\to\infty} (1+3/k)^{k^2/k} = \lim_{k\to\infty} (1+3/k)^k = e^3 > 1$$

Since $r>1$, by the root test $\sum_{k=1}^{\infty} (1+3/k)^{k^2}$ diverges

#2a) $\sum_{k=1}^{\infty} \frac{1}{k^2+8k+12} \approx \sum_{k=1}^{\infty} \frac{1}{k^2}$ use limit comparison

$$L = \lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{1}{k^2+8k+12} \cdot \frac{k^2}{1} = \lim_{k\to\infty} \frac{1}{1+8/k+12/k^2} = 1, \quad 0 < L < \infty$$

Both series are positive

$\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges (p-series, $p=2>1$). So by the limit comparison test, $\sum_{k=1}^{\infty} \frac{1}{k^2+8k+12}$ also converges

b) $\sum_{k=1}^{\infty} \frac{3}{4k+\sqrt{k}}$. Compare to $\sum_{k=1}^{\infty} \frac{1}{k}$. Both series have positive terms. Notice

$$L = \lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{3}{4k+\sqrt{k}} \cdot \frac{k}{1} = \lim_{k\to\infty} \frac{3k}{4k+k^{1/2}} = \lim_{k\to\infty} \frac{3}{4+1/k^{1/2}} = \frac{3}{4}$$

$L = 3/4$ $(0 < L < \infty)$

Now we know $\sum \frac{1}{k}$ diverges (p-series, $p=1$) so by the limit comparison test, the series $\sum_{k=1}^{\infty} \frac{3}{4k+\sqrt{k}}$ diverges

c) We would have use the integral test with partial fractions — ugh!