Day 4 II

a) $\sum_{k=0}^{\infty} \frac{x^k}{k3^k}$ Find radius and interval of convergence

Use ratio test. We know it converges at the center $a=0$.
for $x \neq 1$,

To converge

$$r = \lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left|\frac{x^{k+1}}{(k+1)3^{k+1}} \cdot \frac{k3^k}{x^k}\right| = \lim_{k\to\infty}\left|\frac{kx}{3(k+1)}\right| \overset{HP}{=} \left|\frac{x}{3}\right| < 1$$

We need $|x| < 3$. So $R=3$. Check endpts

At
$x = a - R = 0 - 3 = -3$

using $a_k = \frac{1}{k}$

$$\sum_{k=0}^{\infty} \frac{(-3)^k}{k\,3^k} = \sum_{k=0}^{\infty} \left(\frac{-3}{3}\right)^k \cdot \frac{1}{k} = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{1}{k}$$ Use Alternating Series test. Check two conditions: ① $\lim_{k\to\infty} \frac{1}{k} = 0$ and ② Decreasing: $\frac{1}{k+1} < \frac{1}{k}$

So the series converges at $x = -3$.

At $x = a + R = 0 + 3 = 3$

$$\sum_{k=0}^{\infty} \frac{(3)^k}{k\,3^k} = \sum_{k=0}^{\infty} \frac{1}{k}$$ p-series $p = 1 \leq 1$ Diverges at $x = 3$

include, not included

$R = 3$; Interval $[-3, 3)$

b) $\sum_{n=0}^{\infty} \frac{3^n x^{n+1}}{(2n)!}$. Use ratio test extension. Converges at $a=0$
For $x \neq 0$

Always converges!

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{3^{n+1}x^{n+2}}{(2n+2)!} \cdot \frac{(2n)!}{3^n x^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{3x}{(2n+1)(2n+2)}\right| = 0 < 1$$

The series converges for all x, so $R = \infty$
Interval: $(-\infty, \infty)$

c) $\sum_{k=0}^{\infty} k!(x+4)^k$. Use ratio test extension. Converges at ctr $a=-4$. When $x \neq -4$

Always diverges

$$r = \lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{(k+1)!(x+4)^{k+1}}{k!(x+4)}\right| = \lim_{k\to\infty}\left|(k+1)x\right| = \infty > 1$$

So the Radius $R = 0$. The series converges only at the center $a = -4$.

d) $\sum_{k=0}^{\infty} \frac{5(x-2)^k}{2^k}$ Use ratio test ext- converges at ctr $a=2$ when $x \neq 2$

$r = \lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{5(x-2)^{k+1}}{2^{k+1}} \cdot \frac{2^k}{5(x-2)^k}\right| = \lim_{k\to\infty}\left|\frac{x-2}{2}\right| < 1$ To converge

So $|x-2| < 2$ $\boxed{R=2}$ .... Check endpts

At $x = a-R = 2-2=0$

$\sum_{k=0}^{\infty} \frac{5(0-2)^k}{2^k} = \sum_{k=0}^{\infty} 5\left(\frac{-2}{2}\right)^k = \sum_{k=0}^{\infty} 5(-1)^k$; Use Geometric Series Test $|r| = 1 \geq 1$ Diverges at $x=0$

At $x = a+R = 2+2=4$

$\sum_{k=0}^{\infty} \frac{5(4-2)^k}{2^k} = \sum_{k=0}^{\infty} 5$ Diverges by n^{th} term test $\lim_{n\to\infty} a_n = \lim_{n\to\infty} 5 = 5 \neq 0$.

Diverges at $x=4$

Radius $R=4$; Interval $(0, 4)$

e) $\sum_{n=0}^{\infty} \frac{(x-3)^{2n}}{(-4)^n n}$ Use Ratio Test extension. Converges at ctr $x=3$ when $x \neq 3$

$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-3)^{2n+2}}{(-4)^{n+1}(n+1)} \cdot \frac{(-4)^n n}{(x-3)^{2n}}\right| = \lim_{n\to\infty}\left|\frac{(x-3)^2 n}{(-4)(n+1)}\right|$

$\overset{HP}{=} \lim_{n\to\infty}\left|\frac{(x-3)^2}{4}\right| < 1$ so $|(x-3)^2| < 4$ or $|x-3| < 2 = R$

Check endpts: at $x = a-R = 3-2 = 1$

$\sum_{k=0}^{\infty} \frac{(1-3)^{2k}}{(-4)^k k} = \sum_{k=0}^{\infty} \frac{[(-2)^2]^k}{(-4)^k k} = \sum_{k=0}^{\infty} \frac{4^k}{(-4)^k k} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k}$ $\left(\frac{4}{-4}\right)^k = (-1)^k$ Converges by Alternating Series Test (see part (a)) Converges at $x=1$

At $x = a+R = 3+2=5$

$\sum_{k=0}^{\infty} \frac{(5-3)^{2k}}{(-4)^k k} = \sum_{k=1}^{\infty} \frac{[2^2]^k}{(-4)^k k} = \sum \frac{(-1)^k}{k}$ $\left(\frac{4}{-4}\right)^k$ Converges again by AST converges at $x=5$

So $R = 2$ and interval $= [1, 5]$ includes both endpts