My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

☕ *Practice*

Finish reading 6.4 about the Shell Method to calculate volumes. We will concentrate on rotations around the y-axis, but will illustrate the idea with additional examples about the x-axis. Begin reading Section 6.5 on Length of Curves.

☞ *Take home message.*

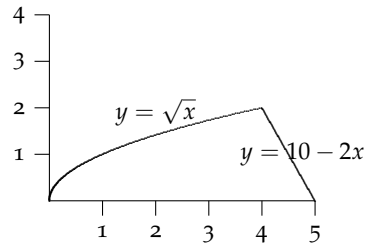
With the disk method, when the curve is rotated about the y-axis, you need to write the integral in terms of y. This means solving for x in terms of y.

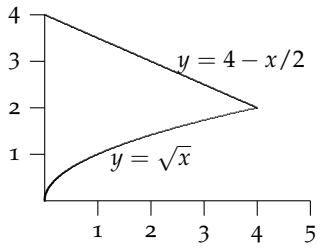
In the same situation, the shell method allows us to use functions of x to determine the volume when rotating around the y axis. We do not have to solve for x. This is an advantage, though sometimes the integrals are harder.

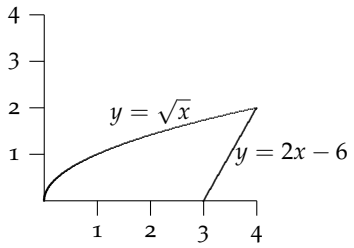
1. **Disk Method Practice.** Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis.
   (*a*) $y = x^2$, $x = 0$, $x = 2$, $y = 0$. (Answer: $32\pi/5$)
   (*b*) $y = 1/x$, $x = 1$, $x = 4$, $y = 0$. (Answer: $3\pi/4$)
   (*c*) $y = 9 - x^2$, $y = 0$. (Answer: $1296\pi/5$)
   (*d*) $y = x^2$, $y = 4x$. (Answer: $2048\pi/15$)
   (*e*) $y = \sqrt{x}$, $y = x$. (Answer: $\pi/6$)
2. **Shell Method Practice.** Page 442–443 #1, 3, 5, 7, 13(shells make this easy), 15.

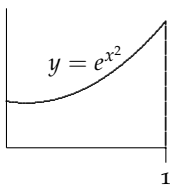
Class Work: Setting it Up

For each of the regions below, use the **shell method** to set up the integrals for the volume of rotation assuming the region is rotated around the y-axis. Can you do the integration for all of these? Explain.









For the first and last regions, use the **disk method** to set up the integrals for the volume of rotation assuming the region is rotated around the y-axis. Can you do the integration for both of these? Explain

Hand In—See Back of Page

The answers are in this list: $8\pi/3$, $16\pi/3$, $8\pi/15$, $32\pi/15$, $11\pi/6$, $5\pi/6$, 8π. **Be neat and put a box around your answers so I can find them.**

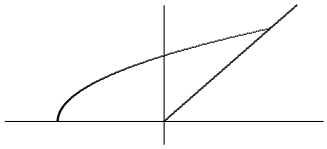
1. Draw the region R in the first quadrant enclosed by $y = x^2$, $y = 2 - x$ and the x-axis.

(*a*) (Disks) Rotate R about the x-axis and find the resulting volume. [Is it the sum of two pieces or outside minus inside?]

(*b*) (Disks) Rotate R about the y-axis and find the resulting volume. [Is it the sum of two pieces or outside minus inside?]

(*c*) (Shells) Rotate R about the y-axis and and **just set up** the integral for finding the resulting volume.

2. Let R be the region in the upper half-plane bounded by $y = \sqrt{x+2}$, the x axis and the line $y = x$. Find the volume resulting when R is rotated around the x axis. Remember: Outside $-$ Inside. How many pieces do you need?



3. Let S be the region in the first quadrant enclosed by $y = x^2$, $y = 2 - x$ and the y-axis. Note: The region is different than in problem 1.

(*a*) Rotate S about the y-axis. Find the volume using the **shell** method. [Is it the sum of two pieces or a difference?]

(*b*) Rotate S about the y-axis. **Just set up the integral** using the disk method. [Is it the sum of two pieces or outside minus inside?]

4. (*a*) **Just carefully set up the integrals using the shell method** for the volume problems in parts (a), (e), and (f) below. You should try to simplify the integrands where possible.

(*b*) Now actually find the volume for part (e). Be careful. What integration method must you use?

Using the Shell Method. Use these diagrams and descriptions for problem 4 above and for the WeBWorK Day 16 problems. **Just set up the integrals** for each of the following volume problems. You should try to simplify the integrands where possible.

(*a*) R is the region enclosed by $y = x^2$, $y = x + 2$, and the y axis in the first quadrant. Rotate R about the y-axis.

(*b*) S is the region enclosed by $y = -x^2 + 2x + 3$, $y = 3x - 3$, the y axis, and the x axis in the first quadrant. Rotate S about the y-axis.

(*c*) T is the region enclosed by $y = -x^2 + 2x + 3$, $y = 3x - 3$, and the x axis in the first quadrant. Rotate T about the y-axis.

(*d*) S is the region enclosed by $y = \frac{1}{2}x^2 + 2$ and $y = x^2$ in the first quadrant. Rotate S about the y-axis.

(*e*) T is enclosed by $y = \sqrt{x-2}$, $y = 4 - x$, the y-axis and the x-axis. Rotate T about the y-axis.

(*f*) V is the region enclosed by $y = \sqrt{x-2}$, $y = 4 - x$, and the x-axis. Rotate V about the y-axis.