My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

Practice

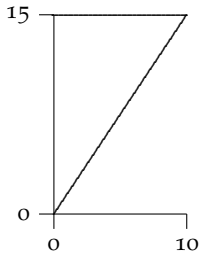
Review Section 6.7 on Physical Applications (pages 460 through Example 4 on page 465.) We will only cover work (lifting) problems. Skip to Section 7.1 and read about **integration by parts** which reverses the product rule. We will spend several classes on techniques of integration.

1. Carefully review Examples 3 and 4 on pages 463–64. Look in the online notes for more examples.
2. Note: Joules are used as the work unit in these problems. See page 463. Try page 468 #27, 29, 31, and 33.
3. Try any problems on the back of the sheet that we do not cover today.

Hand In

Potential Answers for the following problems. $133120\pi/3$ ft-lbs, 48114π ft-lbs, $7904\pi/3$ ft. lbs, $500\pi/3$ ft-lbs, 101250π ft-lbs, $26624\pi/3$ ft. lbs, 7904π ft. lbs, $896\pi/15$ ft-lbs, $2000/3$ ft-lbs

1. (*a*) A cone-shaped reservoir has a 10 foot radius across the top and a 15 foot depth. If the reservoir has 9 feet of oil (density 54 lbs/ft^3) in it, how much work is required to empty it by bringing the oil to the top of the reservoir? (Hint: First determine the equation of the line that determines the cone.)



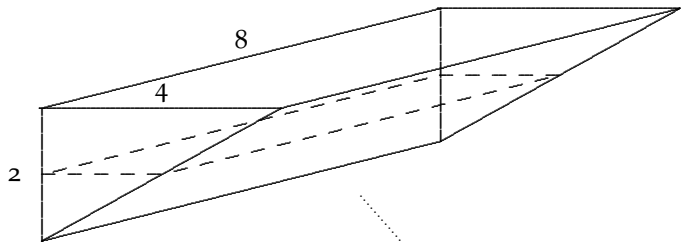
   (*b*) Same question with the reservoir being completely full.

2. (*a*) A pointy-shaped tank is obtained by rotating the region enclosed by the curve $y = \sqrt{x}$ the y-axis and the line $y = 4$. Assume the tank is full of 'heavy' water (density 65 lbs/ft^3). How much work is done in emptying a full tank by removing the water over the tank's top edge?

   (*b*) How much work would be done in emptying only the top 2 feet of water over the edge?

   (*c*) Just **set up** the integral for the work done to pump a full tank 3 feet above the top of the tank.

3. A triangular trough for cattle is 8 ft long. The ends are right triangles with with a 'base' of 4 ft and a height of 2 ft (but the vertex points down, see below). Find the work done by the cattle in emptying just the top foot of water (density 62.5 lbs/ft^3) over the edge. Hint: The cross-sections are rectangles. Use the triangle to find the equation of the line forming the 'hypotenuse' of the triangle on the end.



4. If we get this far: Use integration by parts to determine $\int x \ln x \, dx$. The function you cannot integrate must be u.

Work Problems for Class Today and Practice

Work Formula for Emptying a Tank. Assume the cross-sectional area $A(y)$ of a tank is a continuous function of the height y and that the density of the contents is a constant D. If the contents of the tank to be moved lie in the interval $[a, b]$, then the **work** done to move this material to a height H is

$$\text{Work} = D \int_a^b A(y)[H - y]\, dy.$$

Caution: The tank may not be full, the contents may be moved to a height H above the tank, or the entire tank may not be emptied. If the tank is being *filled* from a source at height H (either at the bottom of or below the tank), then the contents must be moved to each layer height y between a and b so the distance moved is $y - H$ rather than $H - y$.

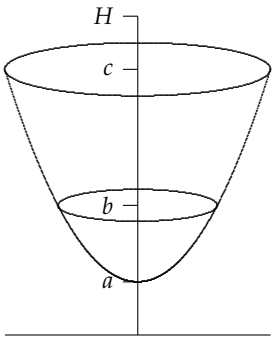


Figure 1: A tank containing a liquid between levels a and b. The top of the tank is at height c. The liquid is to be moved to a height H above the tank.

1. A college pool is rectangular (25 by 75 feet) in shape and is 6 feet deep. The depth of water (density 62.5 lbs/ft^3) is 5 feet. How much work is required to empty the pool by raising the water to the top edge? (Ans: 2050781.25 ft-lbs)

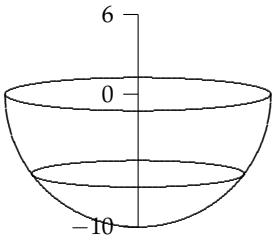
2. A cup shaped tank is obtained by rotating the curve $y = x^2$ about the y-axis where $0 \leq x \leq 3$.

(*a*) Assume the tank is full of 'heavy' water (density 65 lbs/ft^3). How much work is done in emptying the tank by removing the water over the top edge of the tank? (Ans: 7897.5π ft. lbs.)

(*b*) How much work would be done in raising the water 3 feet above the tank's top? (Ans: $15,975\pi$ ft. lbs.)

(*c*) Suppose the tank is empty and is **filled** from a hole in the bottom to a depth of 3 feet. Find the work done. (Ans: 585π ft-lbs.)

3. An underground hemispherical tank with radius 10 ft is filled with oil of density 50 lbs/ft^3. Find the work done pumping the oil to the surface if the top of the tank is 6 feet below ground. It will be easiest to set up the equation of the hemisphere if we think of the top of the tank at height 0 and then pump the oil to a height of 6 feet.



4. A cup shaped tank is obtained by rotating the curve $y = x^3$ about the y-axis where $0 \leq x \leq 2$.

(*a*) Assume the tank is full of water (density 62.5 lbs/ft^3?). How much work is done in emptying the tank by removing the water over the top edge of the tank? (Ans: 3600π ft-lbs?)

(*b*) How much work would be done in raising the water 2 feet above the tank's top? (Ans: 6000π ft-lbs?)

(*c*) Suppose the depth of the liquid in the tank is 1 foot. Find the work required to pump the liquid to the top edge of the tank.