My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

☕ *Practice*

Re-read Section 7.3 on trig integrals. Review the material on this sheet. Do **lots of practice** before and after class, or else this material will rapidly become very confusing. *Read today's online notes for more examples.* **Begin Section 7.4 on Triangle Substitution**

1. Try page 466 # 1, 3, 5, 9–21 (odd), 25, and 29.

Reference: Summary of Trig Integrals. ***Memorize these.***

☞ *Take-Home Message.* OK, the goal here is to be able to use these guidelines for products of trig functions. Here's what you must be able to do without looking back to the guidelines:

1. You need to KNOW the four key identities.
2. You need to be able to integrate low powers ($n = 1$ or 2) of the four main trig functions.
3. You need to be able to integrate products of the form $\sin^n x \cos^m x$ when at least one of the powers is an odd positive integer by splitting off an appropriate factor, using a key trig id, and then using u-substitution (Guidelines 1 and 2 for Products of Sines and Cosines).

1. Four Key Identities Used in Integration.

(*a*) $\cos^2 x + \sin^2 x = 1$

(*b*) $1 + \tan^2 x = \sec^2 x$

(*c*) $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

(*d*) $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

2. Low Powers of the Tangent and Secant Functions. Use trig identities.

(*a*) $\int \tan u \, du = \int \frac{\sin u}{\cos u} \, du = \ln|\sec u| + c.$

(*b*) $\int \tan^2 u \, du = \int \sec^2 u - 1 \, du = \tan u - u + c.$

(*c*) $\int \sec u \, du = \ln|\sec u + \tan u| + c.$

(*d*) $\int \sec^2 u \, du = \tan u + c.$

Hand In

1. (*a*) Fill in the table on the left of values for the sine and cosine functions. Use exact values, not decimal approximations.

(*b*) Fill in the right table using the half-angle formulas.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$					
$\cos\theta$					

$\cos^2(3\theta)$	
$\sin^2(4\theta)$	
$\cos^2(-2\theta)$	

2. Determine the following antiderivatives. All of these are WeBWorK Day 22 Pre-lab problems, so check your answers.

(*a*) $\int \sin^2(4t) \, dt$ via a half-angle trig identity formula .

(*b*) $\int \cos^5(3x) \, dx$ via a reduction formula. Remember to convert to u. (See back.)

(*c*) $\int \sin^2(4x)\cos^3(4x) \, dx$ via a trig identity. Remember to convert to u. (See back.)

(*d*) $\int \sin^3(2x)[\cos(2x)]^{-4} \, dx$ via a trig identity. Remember to convert to u.

(*e*) $\int \sin^2(5x)\cos^2(5x) \, dx$ using Guideline #3 on the back. Your WeBWorK problem may be slightly different.

Reduction Formulas for Large Powers: These are verified using integration by parts. Repeated application may be necessary.

(1) $\displaystyle\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du$

(2) $\displaystyle\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$

(3) $\displaystyle\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$

(4) $\displaystyle\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

Guidelines for Products of Sines and Cosines:

These general principles can help you solve integrals of the form $\int \sin^m x \cos^n x\,dx$.

1. If the power of sine is odd and positive, split off a factor of sine for du and convert the rest to cosines, let $u = \cos x$, and then integrate. For example,

$$\int \overbrace{\sin^{2k+1} x}^{m=2k+1 \text{ odd}} \cos^n x\,dx = \int \overbrace{(\sin^2 x)^k}^{\text{convert to cosines}} \cos^n x \cdot \sin x\,dx = \int \overbrace{(1-\cos^2 x)^k \cos^n x}^{\text{use } u=\cos x} \cdot \overbrace{\sin x\,dx}^{\text{use for } du} = -\int (1-u^2)^k u^n\,du.$$

2. If the power of cosine is odd and positive (and the power of sine is even), split off a factor of cosine for du and convert the rest to sines, let $u = \sin x$, and then integrate. For example,

$$\int \sin^m x \overbrace{\cos^{2k+1} x}^{n=2k+1 \text{ odd}} dx = \int \sin^m x \overbrace{(\cos^2 x)^k}^{\text{convert to sines}} \cos x\,dx = \int \sin^m x(1-\sin^2 x)^k \cdot \overbrace{\cos x\,dx}^{\text{use for } du} = \int u^m (1-u^2)^k\,du.$$

3. If both powers of sine and cosine are *even* and non-negative, make repeated use of the identities $\sin^2 u = \frac{1}{2} - \frac{1}{2}\cos 2u$ and $\cos^2 u = \frac{1}{2} + \frac{1}{2}\cos 2u$ to powers of cosines. Then use reduction formula #1.

4. Use a table of integrals or *WolframAlpha* or other software. Certainly you should use this tool in later courses whether in math or other departments.

Guidelines for Products of Tangents and Secants:

These general principles can help you solve integrals of the form $\int \tan^m x \sec^n x\,dx$.

1. If the power of secant is *even* and positive, split off $\sec^2 x$ to use for du and convert the rest to tangents, then let $u = \tan x$, and integrate. For example,

$$\int \tan^m x \overbrace{\sec^{2k} x}^{n=2k \text{ even}} dx = \int \tan^m x \overbrace{(\sec^2 x)^{k-1}}^{\text{convert to tangents}} \sec^2 x\,dx = \int \tan^m x(1+\tan^2 x)^{k-1} \overbrace{\sec^2 x\,dx}^{\text{use for } du} = \int u^m (1+u^2)^{k-1}\,du.$$

2. If the power of tangent is odd and positive (and the power of secant is odd), split off $\sec x \tan x$ for du and convert the rest to secants, let $u = \sec x$, and then integrate. For example,

$$\int \overbrace{\tan^{2k+1} x}^{m=2k+1 \text{ odd}} \sec^n x\,dx = \int \overbrace{(\tan^2 x)^k}^{\text{convert to secants}} \sec^{n-1} x \cdot \sec x \tan x\,dx = \int (\sec^2 x - 1)^k \sec^{n-1} x \cdot \overbrace{\sec x \tan x\,dx}^{\text{use for } du} = \int (u^2-1)^k u^{n-1}\,du.$$

3. If m is even and n is odd, convert the tangents to secants and use the reduction formula above:

$$\int \overbrace{\tan^{2k} x}^{m=2k \text{ even}} \overbrace{\sec^n x}^{n \text{ odd}} dx = \int \overbrace{(\sec^2 x - 1)^k}^{\text{converted to secant}} \sec^n x\,dx.$$

4. In real life, use *WolframAlpha*, or look in a table of integrals.