My Office Hours: M & W 2:30-4:00, Tu 2:00-3:30, & F 1:30-2:30 or by appointment. Math Intern: Sun: 2:00-5:00, 7:00-10pm; Mon thru Thu: 3:00-5:30 and 7:00-10:30pm in Lansing 310. Website: http://math.hws.edu/~mitchell/Math131F15/index.html.

🛎 Practice

Review all of Chapter 7.8 on improper integrals which we will finish next time. Also see the online notes.

1. Try page 578 #5, 7, 11, 17, 23, and 27. If we get this far: #35, 37, and 43.

Hand In AT LAB tomorrow

Use correct notation with limits! Papers with incorrect notation will be marked down. Potential answers: Diverges $(-\infty)$, -6/5, -3/2, -1, 0, 1/2, 2/3, 1, 3/2, 6/5, Diverges $(+\infty)$.

- o. Work on WeBWorK Day28-15 and finish Day27.
- **1.** Page 578 #6. Use correct limit notation. Similar to a WeBWorK Day 27 problem.
- 2. Page 578 #10. Use correct limit notation.
- **3.** Determine $\int_0^\infty 2xe^{-x^2} dx$. Use correct limit notation.
- **4.** Determine $\int_{1}^{\infty} \frac{4}{1+x^2} dx$. Use correct limit notation.
- **5.** OK, here's an easy one. The *p*-power theorem will be very important. Use it to determine these improper integrals. Do not do any integration. Just use the theorem. Just give the answers. You should not be doing any integration.

(a)
$$\int_1^\infty \frac{1}{x^3} dx$$
 (b) $\int_1^\infty \frac{1}{x^{1/3}} dx$ (c) $\int_1^\infty \frac{1}{x} dx$ (d) $\int_1^\infty \frac{1}{x^{5/3}} dx$

6. Rational Functions and Limits at $\pm \infty$ In Calculus I you dealt with limits such as the following by dividing the numerator and denominator by the highest power of x in the denominator. Remember that this only works when $x \to \infty$ or $x \to -\infty$. See pages 91–94 in your text. [We did this when evaluating Riemann Sums.] Here are two examples:

(A)
$$\lim_{x \to \infty} \frac{6x^2 + x}{x^3 + 1} \stackrel{\text{by }}{=} x^3 \lim_{x \to \infty} \frac{\frac{6}{x} + \frac{1}{x^2}}{1 + \frac{1}{x^3}} = \frac{0}{1}$$
 (B)
$$\lim_{x \to -\infty} \frac{6x^4 + 2x^2}{5x^4 - 1} \stackrel{\text{by }}{=} x^4 \lim_{x \to -\infty} \frac{6 + \frac{2}{x^2}}{5 + \frac{1}{x^4}} = \frac{6}{5}$$

$$\blacksquare$$
 Please turn over

Now here's a trickier situation: Using that $\sqrt{x^6} = x^3$ when *x* is positive. In the second when $x \to -\infty$, we use $\sqrt{x^6} = -x^3$ which is positive because *x* is negative.

(C)
$$\lim_{x \to \infty} \frac{11x^3 + 8x}{\sqrt{16x^6 + x^2}} \stackrel{\text{:}}{=} \lim_{x \to \infty} \frac{\frac{11}{x^3} + \frac{8x}{x^3}}{\sqrt{\frac{16x^6}{x^6} + \frac{x^2}{x^6}}} = \lim_{x \to \infty} \frac{11 + \frac{8}{x^2}}{\sqrt{16 + \frac{1}{x^4}}} = \frac{11}{4}$$

(D)
$$\lim_{x \to -\infty} \frac{11x^3 + 8x}{\sqrt{16x^6 + x^2}} \stackrel{\text{:}}{=} \lim_{x \to \infty} \frac{\frac{11}{x^3} + \frac{8x}{-x^3}}{\sqrt{\frac{16x^6}{x^6} + \frac{x^2}{x^6}}} \lim_{x \to \infty} \frac{-11 - \frac{8}{x^2}}{\sqrt{16 + \frac{1}{x^4}}} = -\frac{11}{4}$$

Now try these without using l'Hôpital's rule.

(a)
$$\lim_{x \to \infty} \frac{6x^8 + x}{4x^8 + 1}$$
 (b) $\lim_{x \to -\infty} \frac{6x^3 + 2x}{5x^2 - 1}$ (c) $\lim_{x \to -\infty} \frac{6x^5}{\sqrt{25x^{10} - 11}}$

7. Bonus: Determine $\int_8^\infty \frac{-2}{x^2 - 6x + 8} dx$. Use correct limit notation.