My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

☕ *Practice*

Read 8.1 and 8.2. The rest of the term will be spent working in Chapters 8 and 9 on sequences and series. This is fun stuff! Some very interesting ideas. But much of the terminology will be new to you so you really need to stay on top of this material.

1. These practice problems familiarize you with sequence terminology.

(*a*) Try page 604 Review Questions #1 through 6. These are very good ones!

(*b*) These will help you get familiar to sequences. Page 604ff #9, 11, 13, 17, 19, 23(a,c), and 27(a,c).

(*c*) These have you calculating limits of sequences. Page 616 # 9, 11, 17, 19, 25, 27, 29, 31, 45, 47, 49, 50, 51.

2. Review: Determine these three integrals; for one use a theorem to make it quick. (Potential Answers: Diverge, 1, 2, 1/3, 1/4).

$$(a)\ \int_0^\infty \frac{x}{(1+x^2)^{3/2}}\,dx \qquad (b)\ \int_1^\infty \frac{1}{x^4}\,dx \qquad (c)\ \int_0^1 \frac{2}{2x-x^2}$$

3. Evaluate the following using highest powers: $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{16x^2-x+6}}{4-2x}$

Summary of Key Limits

You should know and be able to use all of the following limits.

1. $\displaystyle\lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k$. In particular $\displaystyle\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

2. $\displaystyle\lim_{n\to\infty} n^{1/n} = \lim_{n\to\infty}\sqrt[n]{n} = 1$.

3. $\displaystyle\lim_{n\to\infty}\frac{n!}{n^n} = 0$ and $\displaystyle\lim_{n\to\infty}\frac{n^n}{n!} = \infty$ (diverges).

4. Consider the sequence $\{r^n\}_{n=1}^\infty$, where r is a real number.

(*a*) If $|r| < 1$, then $\displaystyle\lim_{n\to\infty} r^n = 0$;

(*b*) If $r = 1$, then $\displaystyle\lim_{n\to\infty} r^n = 1$;

(*c*) Otherwise ($|r| > 1$ or $r = -1$), we have $\displaystyle\lim_{n\to\infty} r^n$ does not exist (diverges).

Hand In

Work on `WeBWorK Day 30` due Thursday. Finish `WeBWorK Day 29B`.

1. (*a*) Page 604 #12

(*b*) Page 604 #14

(*c*) Page 604 #22. (The next four terms up to a_5.)

(*d*) Find an explicit formula for for the general nth term a_n of the sequence $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\}$.

(e) Find an explicit formula for for the general nth term a_n of the sequence $\{\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \dots\}$. Look at the pattern of numbers in the numerator and denominator.

(f) Find a **recurrence** formula for for the general nth term a_n of the sequence $\{64, 32, 16, 8, 4, \dots\}$. Express a_{n+1} in terms of the previous term a_n.

2. Simplify each of these expressions that involve factorials.

$$(a)\ \frac{10!}{8!} \qquad (b)\ \frac{5!}{7!} \qquad (c)\ \frac{(n+2)!}{n!} \qquad (d)\ \frac{3^2 \cdot (n-1)!}{(n+1)!} \qquad (e)\ \frac{4^4}{4!}$$

3. Evaluate the limits of the following sequences. Use **Key Limits** (see other side) where appropriate. Use L'Hopital's rule as needed (convert to x.)

$$(a)\ \left\{\left(1+\frac{2}{n}\right)^n\right\}_{n=1}^{\infty} \qquad (b)\ \left\{\left(\frac{1}{n}\right)^{1/n}\right\}_{n=1}^{\infty} \qquad (c)\ \left\{n \sin\left(\tfrac{1}{n}\right)\right\}_{n=1}^{\infty} \qquad (d)\ \left\{\left(\frac{e}{\pi}\right)^n\right\}_{n=1}^{\infty}$$

4. Now find the limit of $\left\{\ln(n) + \ln\left(\sin\left(\frac{1}{n}\right)\right)\right\}_{n=1}^{\infty}$. HINT: Use a log property to simplify the expression and then use earlier work.

5. **Close Reading**. Very carefully read page 608 in the text where the terms **non-increasing**, **non-dcreasing**, **monotonic**, and **bounded** are defined. Now look at the four sequences on the BACK of the sheet of EXAMPLES OF SEQUENCES. Using the graphs list all of the adjectives that appear to apply to each: **non-increasing**, **non-decreasing**, **monotonic**, and **bounded**. If the sequence is bounded, give a bound B which appears to work.

(a) Upper left: $\left\{(1+n^2)^{1/n}\right\}_{n=1}^{\infty}$

(b) Upper right: $\left\{\sum_{k=1}^{n}(-1)^{n+1}\frac{1}{n}\right\}_{n=1}^{\infty}$

(c) Lower left: $\left\{\frac{4^n}{n!}\right\}_{n=1}^{\infty}$

(d) Lower right: $\left\{\frac{1+(-1)^n}{n}\right\}_{n=1}^{\infty}$

(e) We say that a sequence $\{a_n\}$ is **eventually monotonic** if there is some index N so that for all $n \geq N$, $\{a_n\}_{n=N}^{\infty}$ is monotonic. In other words, from the Nth term on the sequence is either non-decreasing or non-increasing. Which of the four sequences above are eventually monotonic? Give the N for those that are. (Monotonic sequences are automatically eventually monotonic.)

6. Review: Evaluate $\int_1^2 \frac{1}{\sqrt{x^2-1}}\,dx$. If it is improper, use correct notation. What integration technique is required?

7. **Extra Credit**. Find the limits of these sequences:

$$(a)\ \left\{\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n}{n^2}\right\}_{n=1}^{\infty} \qquad (b)\ \left\{\int_1^{\infty}\frac{1}{x^p}\,dx\right\}_{p=2}^{\infty}$$

Math 131 Day 30: Examples of Sequences

1. a) Which of these sequences **converge**?(See the next page, too!)

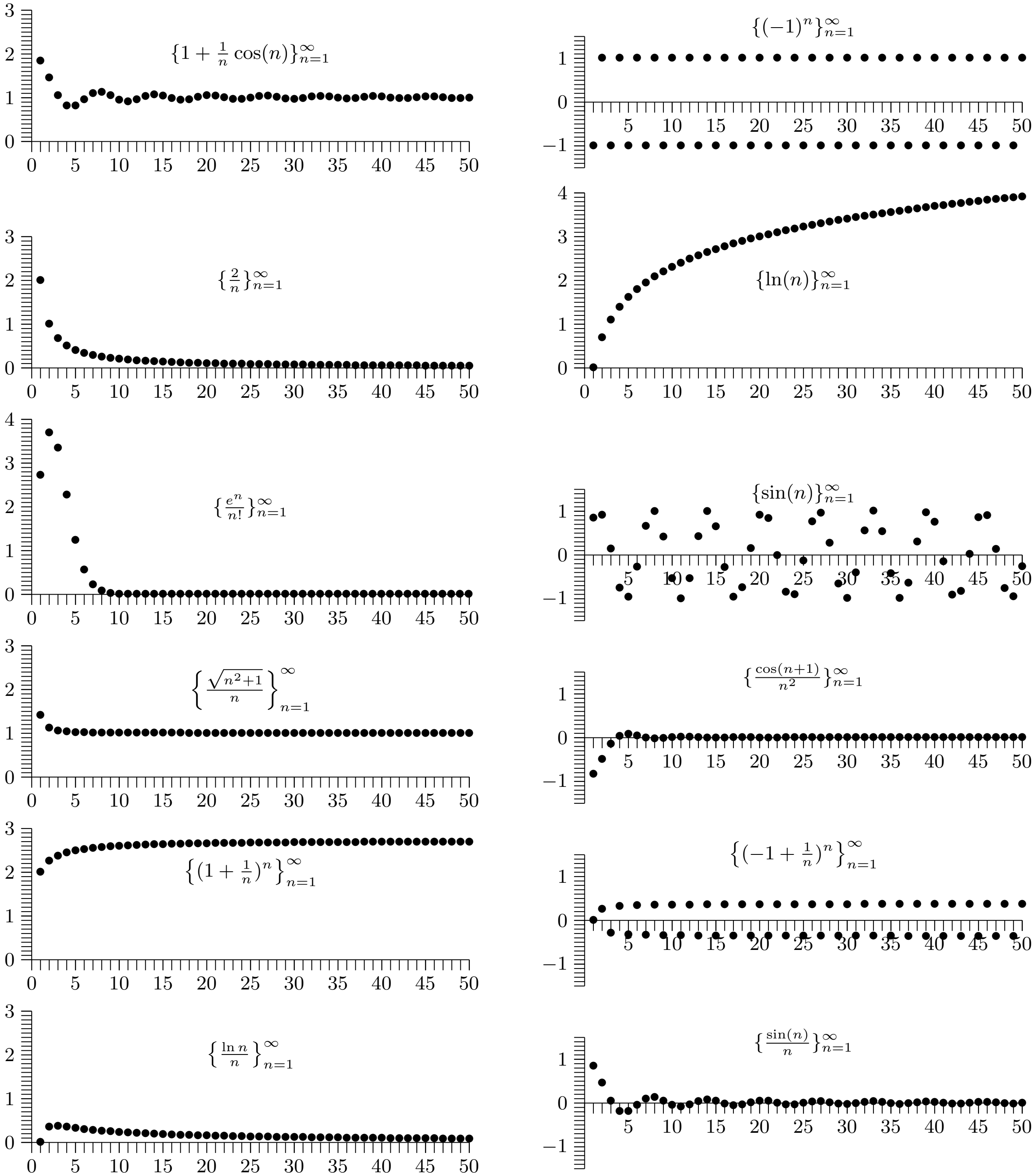
b) Which are **non-decreasing**?

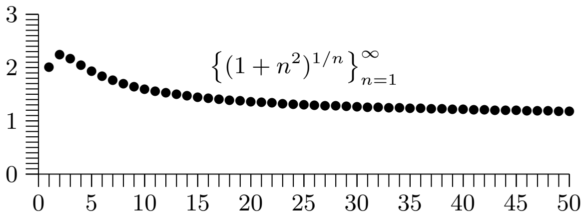
c) Which are **non-increasing**?

d) Which are **monotone**?

e) Which are monotone after the first few terms?

f) Which are **bounded**?

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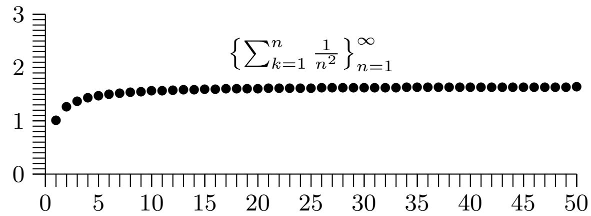
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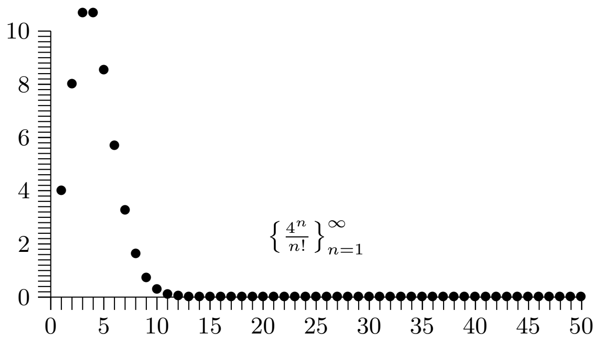
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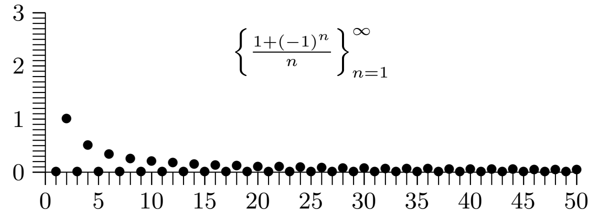
$\left\{\frac{1+(-1)^n}{n}\right\}_{n=1}^{\infty}$