☕ *Practice*

1. Read and review all of Section 8.5 on the ratio, root, and comparison tests.

(*a*) Practice on Ratio Test: Page 647 #9, 11, 13, and 17(esp good)

(*b*) Practice on Root Test (if we get this far): Page 648 #19, 21, 23(good)

Exam Monday. Partial fractions, l'Hôpital's's Rule, improper integrals (both types), sequences (limits, monotone, bounded), series (telescoping, geometric, divergence test, integral test, p-series, ratio test)

Six Tests

1. Ratio Test. Assume that $\sum_{n=1}^{\infty} a_n$ is a series with **positive** terms and let $r = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

(*a*) If $r < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

(*b*) If $r > 1$ or $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > \infty$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

(*c*) If $r = 1$, then the test is inconclusive. (series may converge or diverge).

2. Root Test. Assume that $\sum_{n=1}^{\infty} a_n$ is a series with **positive** terms and let $r = \lim_{n\to\infty} \sqrt[n]{a_n}$.

(*a*) If $r < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

(*b*) If $r > 1$ or $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > \infty$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

(*c*) If $r = 1$, then the test is inconclusive (series may converge or diverge).

3. The Geometric Series Test.

(*a*) If $|r| < 1$, then the geometric series $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$.

(*b*) If $|r| \geq 1$, then the geometric series $\sum_{n=0}^{\infty} ar^n$ diverges.

4. The nth term test for Divergence. If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. (If $\lim_{n\to\infty} a_n = 0$, this test is useless.)

5. The Integral Test. If $f(x)$ is a **positive**, **continuous**, and **decreasing** for $x \geq 1$ and $f(n) = a_n$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

6. The p-series Test. The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $\begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1. \end{cases}$

Hand In At Lab: See Back

Hand In At Lab

Justify your answers with an argument that involves words. Make sure you explain why the series test you use applies and what the conclusion is based on. See the online notes and your text. Four are quick.

1. Page 638 #22
2. Page 638 #24. Note: The interval is $[2, \infty)$ since k starts at 2.
3. Page 647 #10.
4. Page 647 #14.
5. Review: Page 623 #26. Write out the first few terms!
6. Review: Page 638 #16.
7. This is easy: List the letters for three series from the list below for which it would be MOST appropriate to use the Ratio Test. DO NOT DO ANY WORK.

YOU TRY IT 0.1. These are practice—Not to hand in: Pick your method Determine whether the following series converge or diverge. First determine which test to use: divergence (nth term) test, p-series test, integral test, ratio test, or the geometric series test. See if you can quickly determine the test and the result. A complete answer should consist of a little 'argument' (a sentence or two) and any necessary calculations **with appropriate mathematical language**.

$$(a)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^7}} \qquad (b)\ \sum_{n=1}^{\infty} \frac{e^n}{1+e^n} \qquad (c)\ \sum_{n=1}^{\infty} \frac{5^{n+1}}{(2n)!} \qquad (d)\ \sum_{n=1}^{\infty} \frac{2^n+1}{n^2}$$

$$(e)\ \sum_{n=1}^{\infty} \ln(3n+3) - \ln(6n+2) \qquad (f)\ \sum_{n=1}^{\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} \qquad (g)\ \sum_{n=1}^{\infty} 2\left(\frac{-3}{11}\right)^n \qquad (h)\ \sum_{n=2}^{\infty} \frac{n}{\ln n}$$

$$(i)\ \sum_{n=1}^{\infty} 6\left(\frac{5}{4}\right)^n \qquad (j)\ \sum_{n=0}^{\infty} \frac{3n^2}{n^3+1} \qquad (k)\ \sum_{n=1}^{\infty} \frac{(2n+1)!}{(2n-1)!} \qquad (l)\ \sum_{k=1}^{\infty} \frac{3^k}{k^3}$$

$$(m)\ \sum_{n=1}^{\infty} \frac{n^n}{n!} \qquad (n)\ \sum_{n=1}^{\infty} \frac{1}{16+9n^2} \qquad (o)\ \sum_{n=1}^{\infty} \frac{2(n!)}{4^n} \qquad (p)\ \sum_{n=2}^{\infty} \frac{3}{n^2+5n+4}$$

ANSWER TO YOU TRY IT 0.1. C means converges, D means diverges. Your answers should be arguments.

(a) p-series C (b) Divergence Test D (c) Ratio Test C (d) Divergence Test D
(e) Divergence Test D (f) Divergence Test D (g) Geometric Test C (h) Divergence Test D
(i) Geometric Test D (j) Integral Test D (k) Divergence Test D (l) Ratio Test D
(m) Divergence or Ratio Test D (n) Integral Test C (o) Ratio Test D (p) Integral Test C