My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

☕ *Practice*

1. (*a*) **Read Section 9.2** on **power series**. Read the online notes. Read pages 611–614 on Taylor Series.

2. **Vocabulary**: power series, radius of convergence, interval of convergence.

3. Practice with radius and interval of convergence: Try page 609 #3, 7, 9, 11, 13, 15 and 17.

Key Results

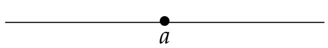
1. **Convergence of Power Series.** For a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ centered at a, precisely one of the following is true.

(*a*) The series converges only at $x = a$. ($R = 0$)

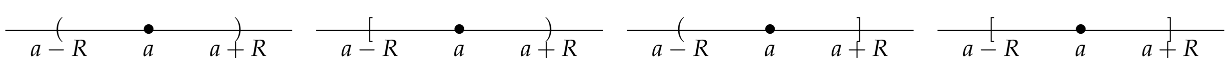
(*b*) There is a real number $R > 0$ so that the series converges absolutely for $|x - a| < R$ and diverges for $|x - a| > R$.

(*c*) The series converges for all x. ($R = \infty$)

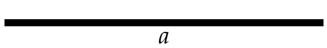
NOTE: In case (b) the power series may converge at both endpoints, either endpoint, or neither endpoint. You have to check the convergence at the endpoints separately. Here's what the intervals of convergence can look like:

(*a*) $R = 0$: 

(*b*) $R \neq 0, \infty$:



$(a-R, a+R)$ $[a-R, a+R)$ $(a-R, a+R]$ $[a-R, a+R]$

(*c*) $R = \infty$: 

Hand In

Be neat. Carefully justify your work. Make this your best assignment.

0. WeBWorK: `Day 40` due Thursday.

1. Finding the Radius and Interval of Convergence. This requires finding the radius of convergence and then checking the endpoints. **Note:** There are lots of similar problems on line in the notes.

(*a*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n2^n}$.

(*b*) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^{n+1}}{(2n)!}$.

(*c*) $\displaystyle\sum_{k=0}^{\infty} k!(x+4)^k$.

(*d*) EZ Bonus: $\displaystyle\sum_{k=1}^{\infty} \frac{k(x-10)^k}{3^k}$.

Over ☞

Answer to Hand In from Friday

See part (b) and how you can show the sequence is decreasing.

1. (*a*) $\sum_{n=1}^{\infty} \dfrac{n^2}{\sqrt{n^6+1}}$. (a) Limit comparison test. (b) $\sum \frac{n^2}{\sqrt{n^6+1}} \approx \sum \frac{n^2}{\sqrt{n^6}} = \sum \frac{n^2}{n^3} = \sum \frac{1}{n}$ (c) The terms are positive.

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2}{\sqrt{n^6+1}} \cdot \frac{n}{1} \overset{\text{HP}}{=} \lim_{n\to\infty} \frac{n3}{\sqrt{n^6}} = \lim_{n\to\infty} \frac{n3}{n^3} = 1.$$

Since $0 < L = 1 < \infty$ and since $\sum \frac{1}{n}$ diverges by the p-series test ($p = 1 \leq 1$), then $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^6+1}}$ diverges by the limit comparison test.

(*b*) (a–c) $\sum_{n=1}^{\infty} \dfrac{(-1)^n n^2}{\sqrt{n^6+1}}$. Use the alternating series test with $a_n = \frac{n^2}{\sqrt{n^6+1}}$. (d) Check the two conditions.

1. $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{n^2}{\sqrt{n^6+1}} \overset{\text{HP}}{=} \lim_{n\to\infty} \dfrac{n^2}{\sqrt{n^6}} \lim_{n\to\infty} \dfrac{n^2}{n^3} = \lim_{n\to\infty} \dfrac{1}{n} = 0.$✓

2. Decreasing? Let $f(x) = \frac{x^2}{\sqrt{x^6+1}}$

$$f'(x) = \frac{2x\sqrt{x^6-1} - \frac{x^2 \cdot 6x^5}{2\sqrt{x^6+1}}}{x^6+1} = \frac{2x\sqrt{x^6-1} - \frac{3x^7}{\sqrt{x^6+1}}}{x^6+1} = \frac{\frac{2x(x^6-1)-3x^7}{\sqrt{x^6+1}}}{x^6+1} = \frac{\frac{2x-x^7}{\sqrt{x^6+1}}}{x^6+1} < 0$$

for $x > 1$. The sequence is decreasing.✓

(e) Since the series satisfies the two hypotheses, by the Alternating Series test, so does $\sum_{n=1}^{\infty} \dfrac{(-1)^n n^2}{\sqrt{n^6+1}}$. converges.