My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

Final Exam: Thursday, December 17, 2015 8:30–11:30 AM, Gulick 206A

This is the exam time is for our lab period in our lab room, Gulick 206A.

1. **The final exam is cumulative**. The material listed below constitutes 90 to 95% of the material on the exam:
   (*a*) Riemann Sums: Drawing upper and lower sums, determining the formula for a simple Riemann sum and computing its limit as $n \to \infty$;
   (*b*) antidifferentiation techniques including substitution, parts (including parts twice), partial fractions (including repeated factors), trig substitutions, low powers of trig functions functions (e.g., $\cos^2 x$, $\sin^2 x$, $\tan^2 x$, $\sec^2 x$). You should be able to do integrals such as $\int \cos^9(4x)\sin^4(4x)\,dx$ by splitting off an odd power. I will give you the reduction formulas for high powers of trig functions, e.g. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$;
   (*c*) relating the graphs of $f(x)$ and $F(x) = \int f(x)\,dx$
   (*d*) applications: area between curves, volumes (including revolutions around the y axis), average value, work, and arc length
   (*e*) L'Hopital's rule including indeterminate forms such as 1^∞, ∞^0, $0 \cdot \infty$;
   (*f*) improper integrals of types: e.g. $\int_a^\infty f(x)\,dx$ or $\int_{-\infty}^a f(x)\,dx$ or $\int_a^b f(x)\,dx$ where f is not defined at one of a or b. Recognizing that an integral $\int_a^b f(x)\,dx$ is improper because it is not defined at some point c in the interval $[a, b]$. Know the p-power theorem for $\int_1^\infty \frac{1}{x^p}\,dx$.
   (*g*) sequences: finding limits, KNOW key limits
   (*h*) series: convergent (divergent) series, partial sums, telescoping, integral test, n-th term test, geometric series test, direct comparison, limit comparison, ratio test, root test, alternating series, absolute and conditional convergence, absolute convergence test, ratio test extension;
   (*i*) power series: finding the radius and interval of convergence;

Review/Practice Session. Monday, December 14 from 10:15 AM to 11:45 AM in Gulick 206A. Look for Practice Questions on line starting today. (Review the earlier practice problems.) Review the Labs, especially Labs 13 and 14. The answers to all the labs and all the homework are on line.

2. Having trouble with a particular topic? Review the online notes.
3. **Do the online practice problems**. The answers will be posted Sunday.
4. **Do the online Practice Exam**. It is a bit longer than the actual exam will be. Try this after you have practiced. The answers will be posted after the Review Session.

In Class

Find the radius and interval of convergence for these power series.

(*a*) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{(-16)^n n}$ (*b*) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-1)^n}{5^{n+1}}$ (*c*) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3x^n}{n!}$ (*d*) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{3n-2}$

(*e*) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2+1}$ (*f*) $\displaystyle\sum_{n=0}^{\infty} (-1)^n n^2 x^n$ (*g*) $\displaystyle\sum_{n=0}^{\infty} 9^n x^{2n}$ (*h*) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{(-16)^{n+1}}$

Day 4 II

a) $\sum_{k=0}^{\infty} \frac{x^k}{k3^k}$ Find radius and interval of convergence

Use ratio test. We know it converges at the center $a=0$.
for $x \neq 1$,

To converge

$$r = \lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left|\frac{x^{k+1}}{(k+1)3^{k+1}} \cdot \frac{k3^k}{x^k}\right| = \lim_{k\to\infty}\left|\frac{kx}{3(k+1)}\right| \overset{HP}{=} \left|\frac{x}{3}\right| < 1$$

We need $|x| < 3$. So $R=3$. Check endpts

At
$x = a - R = 0 - 3 = -3$

using $a_k = \frac{1}{k}$

$$\sum_{k=0}^{\infty} \frac{(-3)^k}{k\,3^k} = \sum_{k=0}^{\infty} \left(\frac{-3}{3}\right)^k \cdot \frac{1}{k} = \sum_{k=0}(-1)^k \cdot \frac{1}{k}$$ Use Alternating Series test, Check

two conditions: ① $\lim_{k\to\infty} \frac{1}{k} = 0$ and ② Decreasing: $\frac{1}{k+1} < \frac{1}{k}$

So the series converges at $x = -3$.

At $x = a+R = 0+3 = 3$

$$\sum_{k=0}^{\infty} \frac{(3)^k}{k\,3^k} = \sum_{k=0}^{\infty} \frac{1}{k}$$ p-series $p = 1 \leq 1$ Diverges at $x = 3$

include, not included

$R = 3$, Interval $[-3, 3)$

b) $\sum_{n=0}^{\infty} \frac{3^n x^{n+1}}{(2n)!}$. Use ratio test extension. Converges at $a=0$
For $x \neq 0$

Always converges!

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{3^{n+1}x^{n+2}}{(2n+2)!} \cdot \frac{(2n)!}{3^n x^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{3x}{(2n+1)(2n+2)}\right| = 0 < 1$$

The series converges for all x, so $R = \infty$
Interval: $(-\infty, \infty)$

c) $\sum_{k=0}^{\infty} k!(x+4)^k$. Use ratio test extension. Converges at ctr $a = -4$. When $x \neq -4$

Always diverges

$$r = \lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{(k+1)!(x+4)^{k+1}}{k!\,(x+4)}\right| = \lim_{k\to\infty}\left|(k+1)x\right| = \infty > 1$$

So the Radius $R = 0$. The series converges only at the center $a = -4$.

d) $\sum_{k=0}^{\infty} \frac{5(x-2)^k}{2^k}$ Use ratio test ext- converges at ctr $a=2$ when $x \neq 2$

To converge

$$r = \lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left| \frac{5(x-2)^{k+1}}{2^{k+1}} \cdot \frac{2^k}{5(x-2)^k} \right| = \lim_{k\to\infty} \left| \frac{x-2}{2} \right| < 1$$

So $|x-2| < 2$ $\boxed{R=2}$ .... Check endpts

At $x = a-R = 2-2 = 0$

$$\sum_{k=0}^{\infty} \frac{5(0-2)^k}{2^k} = \sum_{k=0}^{\infty} 5\left(\frac{-2}{2}\right)^k = \sum_{k=0}^{\infty} 5(-1)^k;$$ Use Geometric Series Test $|r| = 1 \geq 1$ Diverges at $x=0$

At $x = a+R = 2+2 = 4$

$$\sum_{k=0}^{\infty} \frac{5(\cancel{4-2})^k}{\cancel{2^k}} = \sum_{k=0}^{\infty} 5$$ Diverges by n^{th} term test $\lim_{n\to\infty} a_n = \lim_{n\to\infty} 5 = 5 \neq 0$.

Diverges at $x=4$

Radius $R=4$; Interval $(0, 4)$

e) $\sum_{n=0}^{\infty} \frac{(x-3)^{2n}}{(-4)^n n}$ Use Ratio Test extension. Converges at ctr $x=3$ when $x \neq 3$

$$r = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(x-3)^{2n+2}}{(-4)^{n+1}(n+1)} \cdot \frac{(-4)^n n}{(x-3)^{2n}} \right| = \lim_{n\to\infty} \left| \frac{(x-3)^2 n}{(-4)(n+1)} \right|$$

$$\overset{HP}{=} \lim_{n\to\infty} \left| \frac{(x-3)^2}{4} \right| < 1$$ so $|(x-3)^2| < 4$ or $|x-3| < 2 = R$

Check endpts: at $x = a-R = 3-2 = 1$

$$\sum_{k=0}^{\infty} \frac{(1-3)^{2k}}{(-4)^k k} = \sum_{k=0}^{\infty} \frac{[(-2)^2]^k}{(-4)^k k} = \sum_{k=0}^{\infty} \frac{4^k}{(-4)^k k} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k}$$

$\left(\frac{4}{-4}\right)^k = (-1)^k$

Converges by Alternating Series Test (see part (a))

Converges at $x=1$

At $x = a+R = 3+2 = 5$

$\left(\frac{4}{-4}\right)^k$

$$\sum_{k=0}^{\infty} \frac{(5-3)^{2k}}{(-4)^k k} = \sum_{k=1}^{\infty} \frac{[2^2]^k}{(-4)^k k} = \sum \frac{(-1)^k}{k}$$ Converges again by AST

converges at $x=5$

So $R = 2$ and interval $= [1, 5]$ includes both endpts

Math 131 Day 42: Integral Quick Check

	Integral	Technique	Intuition/Reasoning
1	$\int \frac{x}{x^2-4}\,dx$		
2	$\int \frac{x^2-4}{x}\,dx$		
3	$\int \frac{x^2}{1+x^6}\,dx$		
4	$\int xe^x\,dx$		
5	$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$		
7	$\int e^{2x}\cos x\,dx$		
8	$\int x\sec^2 x\,dx$		
9	$\int_{-\pi}^{\pi} x^2\sin x + 8x^3 + \sin x\,dx$		
10	$\int x\ln x\,dx$		
11	$\int \arctan x\,dx$		
12	$\int \tan x\,dx$		
13	$\int x\tan\left(x^2+1\right)\,dx$		
14	$\int \frac{4}{(4-x^2)^{3/2}}\,dx$		
15	$\int \frac{-4x+4}{(x-2)^2x}\,dx$		

	Integral	Technique	Intuition/Reasoning
1	$\int \frac{4}{x^2-4}\,dx$		
2	$\int \frac{x}{x^2-4}\,dx$		
3	$\int \frac{4}{x^2+4}\,dx$		
4	$\int x\sqrt{4-x^2}\,dx$		
5	$\int \frac{\sqrt{x^2-1}}{x}\,dx$		
6	$\int \frac{1}{\sqrt{1-x^2}}\,dx$		
7	$\int \sin^3 x \cos^{4/5} x\,dx$		
8	$\int \sin^3 x \cos^5 x\,dx$		
9	$\int \sin^2 x\,dx$		
9	$\int \sin^2 x \cos^2 x\,dx$		
10	$\int \sec^2 x \tan^{-5} x\,dx$		
11	$\int \sec^4 x \tan^5 x\,dx$		
16	$\int \frac{4x+8}{x^2+4x+5}\,dx$		