Math 131 Lab 3

Make sure to do at least #1 through 8(a) in Lab. Short Answers on back.

1. Assume f is continuous and that $\int_{-4}^{6} f(x)\,dx = 27$, $\int_0^2 f(x)\,dx = 12$, and $\int_2^6 f(x)\,dx = 20$. Evaluate the following. (c) requires a bit of thought!! Use (b) to help with (e).

a) $\displaystyle\int_0^6 f(x)\,dx$ **b)** $\displaystyle\int_{-4}^0 f(x)\,dx$ **c)** $\displaystyle\int_{-1}^1 f(x+1)\,dx$ **d)** $\displaystyle\int_3^3 x^2 f(x)\,dx$ **e)** f_{ave} on $[-4,2]$

2. Evaluate these definite integrals using the Fundamental Theorem of Calculus.

a) $\displaystyle\int_1^4 6x^{1/2} + 2x^{-3/2}\,dx$ **b)** $\displaystyle\int_1^4 \frac{4\sqrt{x}+2}{x}\,dx$ **c)** $\displaystyle\int_1^8 \frac{1}{2\sqrt[3]{x^2}}\,dx$ **d)** $\displaystyle\int_0^{\pi/4} \sec x(\tan x - 2\sec x)\,dx$

3. Evaluate these definite and indefinite integrals. Hint: 'Adjust.'

a) $\displaystyle\int_0^1 4e^{-8t}\,dt$ **b)** $\displaystyle\int \sec(3x)\tan(3x)\,dx$

4. These problems make use of the FTC and FTC II. Begin by determining the given derivatives.

a) $\displaystyle\frac{d}{dx}\left[\int_2^x \cos(t^2+1)\,dt\right]$ **b)** $\displaystyle\frac{d}{dx}\left[\int_x^0 s^3e^s\,ds\right]$ **c)** $\displaystyle\frac{d}{dx}\left[\int_1^{x^3} s\sin(s)\,ds\right]$

d) Suppose that the graph of $f(t)$ passes through the origin and the point $(1,2)$. Evaluate $\int_0^1 f'(t)\,dt$. Notice the integrand is $f'(t)$. So what do you get when you integrate?

e) Suppose that $\int_{-1}^x g(t)\,dt = e^x \arcsin x$. Evaluate $g(0)$ and explain your answer. Hint: First apply FTC to determine g.

f) Is the Fundamental Theorem of Calculus (II) useful when evaluating $\int_1^3 \ln(x^2)\,dx$? Explain.

5. a) Find the average value of $f(t) = 2\sin(\frac{\pi t}{6})$ on $[0,2]$.

b) Let $f(x) = x^{-1}$ on $[1,5]$. The MVTI guarantees that there is a point c in strictly between 1 and 5 where $f(c) = f_{\text{ave}}$. Find the point c. What will you need to determine first?

6. In a day or two we will reverse the chain rule to create a new antiderivative process. To get into 'shape,' find the *derivatives* of the following functions using the chain rule.

a) $f(x) = e^{2\sin x}$ **b)** $f(\phi) = \tan(2\phi^4+3)$ **c)** $g(t) = \ln(3t+1)$ **d)** $p(x) = (x^4+x^2)^6$

7. Now find these antiderivatives. Use the work you did in the previous problem and make an 'adjustment' when needed.

a) $\displaystyle\int 2\cos x e^{2\sin x}\,dx$ **b)** $\displaystyle\int \cos x e^{2\sin x}\,dx$ **c)** $\displaystyle\int 8\phi^3\sec^2(2\phi^4+3)\,d\phi$

d) $\displaystyle\int 24\phi^3\sec^2(2\phi^4+3)\,d\phi$ **e)** $\displaystyle\int \frac{6}{3t+1}\,dt$ **f)** $\displaystyle\int 12(4x^3+2x)(x^4+x^2)^5\,dx$

8. a) Suppose that water is slowly leaking from a still in the Chemistry Department at a rate of $\frac{dv}{dt} = \frac{1}{3t+1}$ gals/hr. How many gallons leak out during the time interval $[1,3]$? Hint: We simply want $v(3) - v(1)$. Translate that into a definite integral and use work you did in the previous problem.

b) Hand in for **extra credit** tomorrow: If you put a 2 gallon plastic bucket under the leak (say at time 0) to catch the water, starting at time 0 how many hours will it take for the bucket to overflow?

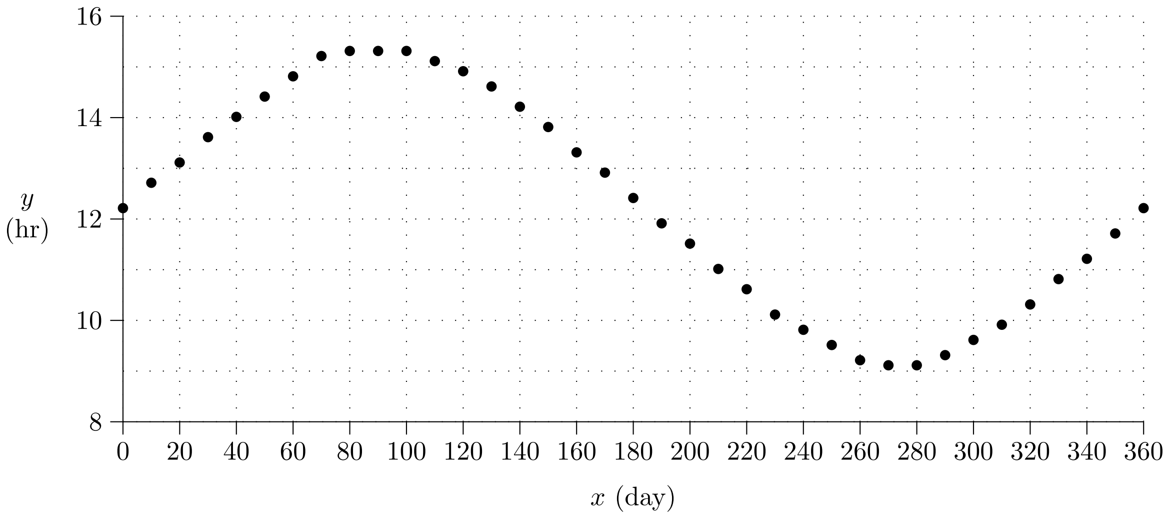
9. a) Suppose the function $s'(t) = 4 - t^2$ represents the velocity in kph of a person walking over the time interval $[0,4]$ hours. How far from her starting point (displacement or net distance travelled) had she walked after 4 hours? Hint: We simply want $s(4) - s(0)$ where $s(t)$ is the position at time t. (Ans: $-16/3$ km.)

b) How many kilometers had she walked (total distance) at the end of 4 hours? Hint: Figure out how many miles she walked forward and how many she walked back by looking at the graph. (Ans: 16 km.)

10. Properties of Integrals. Suppose that $f(x) \geq 0$ on $[0,2]$ and $f(x) \leq 0$ on $[2,5]$ and that $\int_0^2 f(x)\,dx = 6$ and $\int_2^5 f(x)\,dx = -8$. Determine each of the following values. Hint: Try to make a rough sketch of such a function.

a) $\displaystyle\int_0^5 f(x)\,dx$ **b)** $\displaystyle\int_0^5 |f(x)|\,dx$ **c)** $\displaystyle\int_2^5 3|f(x)|\,dx$ **d)** $\displaystyle\int_0^5 (f(x)+|f(x)|)\,dx$

11. Functions as Models. Over the course of a year, the length of day, i.e., the number of hours of daylight, varies. The plot is for Boston, MA with day 0 being March 21. The function $y = f(x) = 3.1\sin\left(\frac{2\pi}{365}x\right) + 12.2$ is a good description of these data where x represents time in days. Find the total amount of light over the course of a year (365 days). (This is the same as the area under the curve below.) Use a mental adjustment.



Extra Fun if You Finish Early

12. a) What is the **derivative** of $F(x) = x\sin x$?

b) Find the **antiderivative** of $f(x) = \sin x + x\cos x$. Hint: Look up.

c) Evaluate $\int_0^{\pi/2} \sin x + x\cos x\,dx$.

d) Find the average value of $\sin x + x\cos x$ on $[0,\pi]$

e) Find the point c strictly between 0 and π where $f(c) = f_{\text{ave}}$.

13. Find the average value of $f(t) = 2t^{99} + \pi t^{103} - 16t^{47} - 17t^3 + 3t^2$ on $[-2,2]$. Why is this EZ? (See page 577 in text.)

Short Answers

Complete answers will be online.

1. 32, -5, 12, 0, $\frac{7}{6}$.

2. 30, $8 + 2\ln 4$, $\frac{3}{2}$, $\sqrt{2} - 3$

3. Multiply by the reciprocal of the constant on the "inside" to reverse the chain rule: $-\frac{1}{2}[e^{-8} - 1]$, $\frac{1}{3}\sec(3x) + c$.

4. a) $\cos(x^2+1)$; (b) $-x^3e^x$, (c) $x^3\sin(x^3)\cdot 3x^2$, (d) 2; (e) 1; (f) 1, No. But see longer answer online.

5. $\frac{3}{\pi}$; $c = \frac{4}{\ln 5}$.

6. Use the chain rule: $\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$. $2\cos x e^{2\sin x}$, $8\phi^3\sec^2(2\phi^4+3)$, $\frac{3}{3t+1}$, $6(4x^3+2x)(x^4+x^2)^5$

7. $e^{2\sin x} + c$, $\frac{1}{2}e^{2\sin x} + c$, $\tan(2\phi^4+3) + c$, $3\tan(2\phi^4+3) + c$, $2\ln|3t+1| + c$, $2(x^4+x^2)^6 + c$

8. Homework

9. $-16/3$ km

10 -2, 14, 24, 12

11. 4,453

13. 4

Math 131 Lab 3 Answers

1. a) $\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx = 12 + 20 = 32.$

b) $\int_{-4}^0 f(x)\,dx = \int_{-4}^6 f(x)\,dx - \int_0^6 f(x)\,dx = 27 - 32 = -5.$

c) $y = f(x+1)$ is a horizontal shift of $y = f(x)$ one unit to the left, so $\int_{-1}^1 f(x+1)\,dx = \int_0^2 f(x)\,dx = 12.$

d) $\int_3^3 x^2 f(x)\,dx = 0$ because the endpoints are the same.

e) $f_{\text{ave}} = \frac{1}{2-(-4)} \int_{-4}^2 f(x)\,dx = \frac{1}{6}\left[\int_{-4}^0 f(x)\,dx + \int_0^2 f(x)\,dx\right] = \frac{1}{6}[-5+12] = \frac{7}{6}$

2. Using the FTC (part 2)

a) $\displaystyle\int_1^4 6x^{1/2} + 2x^{-3/2}\,dx = 4x^{3/2} - 4x^{-1/2}\Big|_1^4 = (32-2) - (4-4) = 30$

b) $\displaystyle\int_1^4 4x^{-1/2} + 2x^{-1}\,dx = 8x^{1/2} + 2\ln|x|\Big|_1^4 = (16 + 2\ln 4) - (8+0) = 8 + 2\ln 4$

c) $\displaystyle = \frac{1}{2}\int_1^8 x^{-2/3}\,dx = \frac{3}{2}x^{1/3}\Big|_1^8 = \frac{3}{2}(2-1) = \frac{3}{2}$

d) $\displaystyle = \int_0^{\pi/4} \sec x\tan x - 2\sec^2 x\,dx = \sec x - 2\tan x\Big|_0^{\pi/4} = (\sqrt{2}-2) - (1-0) = \sqrt{2} - 3$

3. Multiply by the reciprocal of the constant on the "inside" of the function (this reverses the chain rule):

a) $\displaystyle\int_0^1 4e^{-8t}\,dt = -\tfrac{4}{8}e^{-8t}\Big|_0^1 = -\tfrac{1}{2}[e^{-8} - 1]$

b) $\displaystyle\int \sec(3x)\tan(3x)\,dx = \tfrac{1}{3}\sec(3x) + c$

4. FTC (part 1) says: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$. So

a) $\frac{d}{dx}\left[\int_2^x \cos(t^2+1)\,dt\right] = \cos(x^2+1).$

b) $\frac{d}{dx}\left[\int_x^0 s^3 e^s\,ds\right] = \frac{d}{dx}\left[-\int_0^x s^3 e^s\,ds\right] = -x^3 e^x.$

c) The chain rule version: $\frac{d}{dx}\left[\int_a^u f(t)\,dt\right] = f(u)\frac{du}{dx}$. So $\frac{d}{dx}\left[\int_1^{x^3} s\sin(s)\,ds\right] = x^3\sin(x^3)\cdot 3x^2$

d) Since $f(0) =$ and $f(1) = 2$, then $\int_0^1 f'(t)\,dt = f(t)\Big|_0^1 = f(1) - f(0) = 2 - 0 = 2.$

e) FTC (part 1) implies $g(x) = \frac{d}{dx}\left[\int_{-1}^x g(t)\,dt\right] = \frac{d}{dx}[e^x \arcsin x] = e^x\arcsin x + \frac{e^x}{\sqrt{1-x^2}}$. So $g(0) = e^0\arcsin 0 + \frac{e^0}{\sqrt{1-0^2}} = 0 + 1 = 1.$

f) To apply FTC (part 2) you need to know an antiderivative for the integrand. We don't know an antiderivative for $\ln(x^2)$. **Exception:** If you happen to remember Lab 1, you found that $\int \ln x\,dx = x\ln x + c$. So

$$\int_1^3 \ln(x^2)\,dx = \int_1^3 2\ln x\,dx = 2\int_1^3 \ln x\,dx = 2[x\ln x - x]\Big|_1^3 = 2[(\ln 3 - 3) - 1] = 2\ln 3 - 8.$$

5. a) $f_{\text{ave}} = \frac{1}{2-0}\int_0^2 2\sin(\frac{\pi t}{6})\,dt = -\frac{1}{2}\cdot 2\cdot\frac{6}{\pi}\cdot\cos\frac{\pi t}{6}\Big|_0^2 = -\frac{6}{\pi}\left[\cos\frac{\pi}{3} - \cos 0\right] = -\frac{6}{\pi}\left[\frac{1}{2} - 1\right] = \frac{3}{\pi}$

b) $f_{\text{ave}} = \frac{1}{5-1}\int_1^5 x^{-1}\,dx = -\frac{\ln x}{4}\Big|_1^5 = \frac{\ln 5}{4}$. Now find c so that $f(c) = c^{-1} = \frac{1}{c} = \frac{\ln 5}{4}$. This means $c = \frac{4}{\ln 5}$.

6. Use the chain rule: $\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$.

a) $f'(x) = 2\cos x e^{2\sin x}$

b) $f'(\phi) = 8\phi^3\sec^2(2\phi^4 + 3)$

c) $g'(t) = \dfrac{3}{3t+1}$

d) $p'(x) = 6(4x^3 + 2x)(x^4 + x^2)^5$

7. Match these up with the answers for the previous problem and adjust with a constant where needed.

a) $\int 2\cos x e^{2\sin x}\,dx = e^{2\sin x} + c$

b) $\int \cos x e^{2\sin x}\,dx = \frac{1}{2}e^{2\sin x} + c$

c) $\int 8\phi^3 \sec^2(2\phi^4+3)\,d\phi = \tan(2\phi^4+3) + c$

d) $\int 24\phi^3 \sec^2(2\phi^4+3)\,d\phi = 3\tan(2\phi^4+3) + c$

e) $\int \frac{6}{3t+1}\,dt = 2\ln|3t+1| + c$

f) $\int 12(4x^3+2x)(x^4+x^2)^5\,dx = 2(x^4+x^2)^6 + c$

8. For Homework Day 8.

9. a) Net change in position $s(4) - s(0) = \int_0^4 s'(t)\,dt = \int_0^4 4 - t^2\,dt = 4t - \frac{1}{3}t^3\Big|_0^4 = (16 - \frac{64}{3}) - (0) = -\frac{16}{3}$ km.

b) Total distance travelled: Here we need to find the area above the x-axis and below the x-axis and treat them as both positive and add them together. So where is $4-t^2$ negative? $4-t^2 < 0$ when $4 < t^2$ so $2 < t$ (or $t < -2$). So we need $t > 2$. So we have to split the interval $[0,4]$ into two pieces: $[0,2]$ where $s'(t)$ is positive and $[2,4]$ where $s'(t)$ is negative. On the first interval

$$\int_0^2 4 - t^2\,dt = 4t - \frac{1}{3}t^3\Big|_0^2 = (8 - \tfrac{8}{3}) - 0 = \tfrac{24}{3}\text{ km.}$$

On the second interval

$$\int_2^4 4 - t^2\,dt = 4t - \frac{1}{3}t^3\Big|_2^4 = 4t - \frac{1}{3}t^3\Big|_0^4 = (16 - \tfrac{64}{3}) - (8 - \tfrac{8}{3}) = -\tfrac{40}{3}\text{ km.}$$

So the total distance covered is $\frac{24}{3} + \frac{40}{3} = \frac{64}{3}$ km.

10. a) $\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^5 f(x)\,dx = 6 - 8 = -2.$

b) $f(x) \le 0$ only on $[2,5]$ so $|f(x)| = -f(x)$ there. So $\int_0^5 |f(x)|\,dx = \int_0^2 f(x)\,dx + \int_2^5 -f(x)\,dx = 6 + 8 = 14.$

c) $\int_2^5 3|f(x)|\,dx = 3\int_2^5 -f(x)\,dx = 3(8) = 24.$

d) $\int_0^5 (f(x) + |f(x)|)\,dx = \int_0^5 f(x)\,dx + \int_0^5 |f(x)|\,dx = -2 + 14 = 12.$

11. $\int_0^{365} 3.1\sin\left(\frac{2\pi}{365}x\right) + 12.2\,dx = -\frac{365}{2\pi}\cdot 3.1\cdot\cos\left(\frac{2\pi}{365}x\right) + 12.2x\Big|_0^{365}$

$$= \left(-\tfrac{365}{2\pi}\cdot 3.1[\cos(2\pi) - \cos 0]\right) + (12.2\cdot 365 - 0) = \left(-\tfrac{365}{2\pi}[1-1]\right) + 4,453 = 4,453\,\text{hr}$$

12. a) $F'(x) = \sin x + x\cos x$

b) Just look at part (a). The antiderivative of $f(x) = \sin x + x\cos x$ is $F(x) = x\sin x$.

c) $\int_0^{\pi/2} \sin x + x\cos x\,dx = x\sin x\Big|_0^{\pi/2} = \frac{\pi}{2}\cdot 1 - 0 = \frac{\pi}{2}.$

d) $f_{\text{ave}} = \frac{1}{\pi-0}\int_0^{\pi} \sin x + x\cos x\,dx = \frac{1}{\pi}(x\sin x)\Big|_0^{\pi} = \frac{1}{\pi}(\pi\cdot 0 - 0) = 0.$

e) We need $f(c) = c\sin(c) = f_{\text{ave}} = 0$. So $c = \frac{\pi}{2}$ because $c = 0$ is not between the endpoints.

13. Use symmetry, split into odd and even terms: $f_{\text{ave}} = \dfrac{1}{2-(-2)}\displaystyle\int_{-2}^{2} 2t^{99} + \pi t^{103} - 16t^{47} - 17t^3 + 3t^2\,dt$

$$= \frac{1}{4}\int_{-2}^{2} 2t^{99} + \pi t^{103} - 16t^{47} - 17t^3\,dt + \frac{1}{4}\int_{-2}^{2} 3t^2\,dt = 0 + \frac{2}{4}\int_0^2 3t^2\,dt = \frac{1}{2}t^3\Big|_0^2 = 4.$$