# Math 131 Lab 4

**Remember:** If $a > 0$, then $\displaystyle\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\frac{u}{a} + c$ and $\displaystyle\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\frac{u}{a} + c$. Brief answers are on the back. **Complete answers are online.**

**1.** Evaluate these definite integrals; be sure to switch the limits of integration when you do a substitution. Part (b) is one of those "tricky" substitutions calculus teachers put on tests.

**a)** $\displaystyle\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx$ **b)** $\displaystyle\int_{-1}^0 (x-2)\sqrt{x+1}\,dx$ **c)** $\displaystyle\int_{-1}^1 \frac{t}{9+t^4}\,dt$ **d)** $\displaystyle\int_e^{e^2} \frac{1}{t\ln t}\,dt$

**e)** Use your work in part (a) to find the average value of $f(x) = \frac{x}{\sqrt{x^2+9}}$ on $[0, 4]$.

**2.** Here are a few more. For part (c), fill in a value of $n$ that will make this an easy problem to do, then solve the problem. For part (d) use a different value of $n$ and again solve.

**a)** $\displaystyle\int \frac{e^{2x}}{1+e^{4x}}\,dx$ **b)** $\displaystyle\int \frac{\sec^2(3x)}{\sqrt{1-\tan^2(3x)}}\,dx$ **c)** $\displaystyle\int \frac{t^n}{\sqrt{1-t^8}}\,dt$ **d)** $\displaystyle\int \frac{t^n}{\sqrt{1-t^8}}\,dt$

**3.** Two similar integrals (a and b) that are done differently. Only one of these requires substitution. Do you see how to distinguish them? What pattern do you look for when doing substitution? Then try the third.

**a)** $\displaystyle\int x(2+x^3)\,dx$ **b)** $\displaystyle\int x^2(2+x^3)^4\,dx$ **c)** $\displaystyle\int \frac{1}{\sqrt{9-4t^2}}\,dt$

**4.** Suppose that the velocity of an object measured in meters/sec is $v(t) = t^2 - 6t + 8$ for $0 \le t \le 5$.

**a)** Find the displacement (net change in position) over the given interval.

**b)** Find the TOTAL distance traveled over the same interval.

**5.** When records were first kept, the population of a rural town was 250 people ($t = 0$). Since then the population has grown at a rate $P'(t) = 30(1+\sqrt{t})$, where $t$ is measured in years.

**a)** What was the population at time $t = 20$ years?

**b)** Find the population $P(t)$ for an arbitrary time $t \ge 0$.

**6.** Power and energy are often used interchange- ably, but they are quite different. **Energy** is what makes matter move or heat up and is measured in units of joules (J) or Calories (Cal), where 1 Cal = 4184 J. One hour of walking consumes roughly $10^6$ J, or 250 Cal. On the other hand, **power** is the rate at which energy is used and is measured in watts (W; 1W = 1J/s). Other useful units of power are kilowatts (1kW = 1000 W) and megawatts (1MW = $10^6$ W). If energy is used at a rate of 1 kW for 1 hr, the total amount of energy used is 1 kilowatt-hour (kWh). Suppose the power function of a large city over a 24-hr period is given by $P(t) = E'(t) = 300 - 200\sin\left(\frac{\pi t}{12}\right)$ where $P$ is measured in MW and $t$ is measured in hours with $t = 0$ corresponds to 6:00 p.m.

**a)** How much energy is consumed by this city in a typical 24-hr period? Express the answer in MWh.

**b)** How much energy is consumed by this city from midnight to 6 a.m.? (Is it one-fourth of the entire day's consumption?)

**c)** Burning 1 kg of coal produces about 450 kWh of energy. How many kg of coal are required to meet the energy needs of the city for 1 day? For 1 yr?

**d)** Fission of 1 gram of uranium-235 produces about 16,000 kWh of energy. How many grams of uranium are needed to meet the energy needs of the city for 1 day? For 1 yr?

**7.** Determine these antiderivatives. Most are simpler substitutions. Check your answers by differentiating.

**a)** $\displaystyle\int \sec(-8t)\,dt$ **b)** $\displaystyle\int e^{2\sin x}\cos x\,dx$ **c)** $\displaystyle\int \phi^3\sec^2(2\phi^4+3)\,d\phi$ **d)** $\displaystyle\int \frac{6}{3t+1}\,dt$

**e)** $\displaystyle\int \frac{\tan\sqrt{x}}{\sqrt{x}}\,dx$ **f)** $\displaystyle\int \frac{\sec^2 3x}{\tan^4 3x}\,dx$ **g)** $\displaystyle\int \frac{t}{\sqrt{4-16t^4}}\,dt$

**8. a)** Evaluate $\int \tan^2(\pi x)\,dx$. Hint: Use a trig identity.

**b)** Determine $\int \tan(2\pi x)\,dx$.

**c)** Determine $\int xe^{2x^2}\sec(e^{2x^2})\,dx$.

**9.** Three similar integrals.

**a)** $\displaystyle\int \frac{1+x^4}{x}\,dx$ **b)** $\displaystyle\int \frac{x}{1+x^2}\,dx$ **c)** $\displaystyle\int \frac{x^2}{1+x^6}\,dx$

**10. Two Key Identities: Half Angle Formulæ.** You should memorize the following two identities. They will make integrating certain trig functions MUCH easier.

$$\sin^2\theta = \tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta \quad \text{and} \quad \cos^2\theta = \tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta$$

The half angle formulas are used to integrate $\sin^2 u$ or $\cos^2 u$ in the obvious way. Determine

**a)** $\displaystyle\int \cos^2(8x)\,dx$ **b)** $\displaystyle\int \sin^2(\pi x)\,dx$

Challenge for Those Who Finish Early

**11. a)** [Unusual Substitution] Determine $\displaystyle\int x\sqrt[3]{x+1}\,dx$. Try this method: Instead of letting $u = x+1$ which is the inside function, let $u$ be the entire root: $u = \sqrt[3]{x+1}$ and solve for $x$.

**b)** Return to Problem 3(c) and use the method of part (a) above to solve the problem. Do you get the same answer?

**12. a)** [Multi-Substitution] Sometimes you need more than one substitution. Laissez les bons temps rouler! Determine $\displaystyle\int x^2\cos^2(x^3)\sin(x^3)\,dx$. Start with the obvious substitution: $u = x^3$.

**b) Extra Credit**: This one is sweet: Determine $\displaystyle\int \frac{1}{\sqrt{1+\sqrt{1+x}}}\,dx$. Start with the substitution: $u = \sqrt{1+x}$.

**13. More Fun**: (a) A sprinter accelerates during the first four seconds of the 100 meter dash. If $a(t) = 6\sqrt{t}$ m/s$^2$, what is the sprinter's average acceleration during this period? (b) What is the average velocity? (c) **Bonus:** Average position?

**14.** Suppose that $\int_x^{-1} g(t)\,dt = \arctan(3x^2)$. Evaluate $g(1)$ and show your "work." Hint: Apply FTC II.

**Math 131: Short Answers to Lab 4**

**1.** (a) 2. (b) $-\frac{8}{5}$. (c) 0. (d) $\ln 2$ (e) $\frac{1}{2}$.

**2.** (a) $\frac{1}{2}\arctan e^{2x} + c$. (B) $\frac{1}{3}\arcsin(\tan(3x)) + c$. (c) $\frac{1}{4}\arcsin t^4 + c$. (d) $-\frac{1}{4}\sqrt{1-t^8} + c$

**3.** (a) $x^2 + \frac{1}{5}x^5 + c$. (b) $\frac{1}{15}(2+x^3)^5 + c$. (c) $\frac{1}{2}\arcsin\frac{2t}{3} + c$.

**4.** (a) $\frac{20}{3}\ m; (b) frac283$ m.

**5.** (a) 2639 people; (b) $250 + 30\left(x + \frac{2x^{3/2}}{3}\right)$ people.

**6. a)** 7200 MWh; (b) 600 MWh; (c) Day: 1600kg, Yr: 5,840,000kg; (d) Day: 450g, Yr: 164,250 g.

**7.** (a) $-\frac{1}{8}\ln|\sec(-8t)+\tan(-8t)| + c$. (b) $\frac{1}{2}e^{2\sin x} + c$. (c) $\frac{1}{8}\tan(2\phi^4+3) + c$. (d) $2\ln|3t+1| + c$. (e) $2\ln|\sec u| + c = 2\ln|\sec(\sqrt{x})| + c$. (f) $-\frac{1}{9}(\tan 3x)^{-3} + c$. (g) $\frac{1}{8}\arcsin 2t^2 + c$

**8.** (a) $\frac{1}{\pi}\tan(\pi x) - x + c$. (b) $\frac{1}{2\pi}\ln|\sec(2\pi x)| + c$. (c) $\frac{1}{4}\ln|\sec e^{2x^2} + \tan e^{2x^2}| + c$.

**9.** (a) $\ln|x| + \frac{1}{4}x^4 + c$; (b) $\frac{1}{2}\ln(1+x^2) + c$; (c) $\frac{1}{3}\arctan(x^3) + c$.

**10. a)** $\frac{1}{2}x + \frac{1}{32}\sin(16x) + c$. (b) $\frac{1}{2}x + \frac{1}{4\pi}\sin(2\pi x) + c$.

**11. a)** $\frac{3}{7}(x+1)^{7/3} - \frac{3}{4}(+1)^{4/3} + c$. (b) $(x-1)^{3/2} + 2\sqrt{x+1} + c$.

**12. a)** $-\frac{1}{9}\cos^3(x^3) + c$. (b) XC

**13. a)** Ave acc $= 8$ m/s$^2$. (b) Ave vel $= \frac{64}{5}$ m/s. (c) XC

**14.** $g(1) = -\frac{6}{10}$.

## Math 131: Answers to Lab 4

Remember $\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\frac{u}{a} + c$ and $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\frac{u}{a} + c$

**1. a)** $u = x^2+9$, $\frac{1}{2}du = x dx$. Change limits: $x = 0, \Rightarrow u = 5$; $x = 4, \Rightarrow u = 25$. $\frac{1}{2}\int_9^{25} u^{-1/2}\,du = u^{1/2}\Big|_9^{25} = 5-3 = 2$.

**b)** Inside: $u = x+1$, $du = dx$, $x = u-1$, $x-2 = u-3$. Change limits: $x = -1, \Rightarrow u = 0$; $x = 0, \Rightarrow u = 1$. $\int_0^1 (u-3)u^{1/2}\,du = \int_0^1 u^{3/2} - 3u^{1/2}\,du = \frac{2}{5}u^{5/2} - 2u^{3/2}\Big|_0^1 = \frac{2}{5} - 2 = -\frac{8}{5}$.

**b)** Alternative solution. $u = \sqrt{x+1}$, $u^2 = x+1$, $u^2-1 = x$, $2u\,du = dx$, and $x-2 = u^2-3$. Change limits: $x = -1, \Rightarrow u = 0$; $x = 0, \Rightarrow u = 1$. $\int_0^1 (u^2-3)\cdot u\cdot 2u\,du = \int_0^1 2u^4 - 6u^2\,du = \frac{2}{5}u^5 - 2u^3\Big|_0^1 = \frac{2}{5} - 2 = -\frac{8}{5}$.

**c)** $a^2 = 9$, $a = 3$, $u^2 = t^4$, $u = t^2$, $\frac{1}{2}du = tdt$. Change limits: $t = -1, \Rightarrow u = 1$; $t = 1, \Rightarrow u = 1$. Uh, oh! $\frac{1}{2}\int_1^1 \frac{1}{a^2+u^2}\,du = 0$.

**d)** $u = \ln t$, $du = \frac{1}{t}dt$. Change limits: $t = e, \Rightarrow u = \ln e = 1$; $t = e^2, \Rightarrow u = \ln e^2 = 2$. $\int_1^2 \frac{1}{u}\,du = \ln|u|\Big|_1^2 = \ln 2 - \ln 1 = \ln 2$.

**e)** Using part (a), $f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-0}(2) = \frac{1}{2}$.

**2. a)** $u^2 = e^{4x}$, so $u = e^{2x}$, $du = 2e^{2x}\,dx$, $\frac{1}{2}du = e^{2x}\,dx$.

$$\int \frac{e^{2x}}{1+e^{4x}}\,dx = \frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\arctan u + c = \frac{1}{2}\arctan e^{2x} + c$$

**b)** $u^2 = \tan^2(3x)$, so $u = \tan(3x)$, $du = 3\sec^2(3x)\,dx$, $\frac{1}{3}du = \sec^2(3x)\,dx$.

$$\int \frac{\sec^2(3x)}{\sqrt{1-\tan^2(3x)}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{3}\arcsin u + c = \frac{1}{3}\arcsin(\tan(3x)) + c$$

**c)** Use $n = 3$. $u^2 = t^8$, so $u = t^4$, $du = 4t^3\,dt$, $\frac{1}{4}du = t^3\,dt$.

$$\int \frac{t^3}{\sqrt{1-t^8}}\,dt = \frac{1}{4}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{4}\arcsin u + c = \frac{1}{4}\arcsin(t^4) + c$$

**d)** Use $n = 7$. $u = 1-t^8$, $du = -8t^7\,dt$, $-\frac{1}{8}du = t^7\,dt$.

$$\int \frac{t^7}{\sqrt{1-t^8}}\,dt = -\frac{1}{8}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{4}u^{1/2} + c = -\frac{1}{4}\sqrt{1-t^8} + c$$

**3. a)** Multiply out $= \int 2x + x^4\,dx = x^2 + \frac{1}{5}x^5 + c$

**b)** $u = 2+x^3$, $\frac{1}{3}du = x^2 dx$. $\frac{1}{3}\int u^4\,du = \frac{1}{15}u^5 + c = \frac{1}{15}(2+x^3)^5 + c$.

**c)** $a^2 = 9$, so $a = 3$. $u^2 = 4t^2$, so $u = 2t$, $du = 2\,dt$, $\frac{1}{2}du = dt$.

$$\int \frac{1}{\sqrt{9-4t^2}}\,dt = \frac{1}{2}\int \frac{1}{\sqrt{a^2-u^2}}\,du = \tfrac{1}{2}\arcsin\tfrac{u}{a} + c = \tfrac{1}{2}\arcsin\tfrac{2t}{3} + c$$

**4. a)** Displacement $= \int_0^5 t^2 - 6t + 8\,dt = \frac{t^3}{3} - 3t^2 + 8t\Big|_0^5 = \left(\frac{125}{3} - 75 + 40\right) - (0) = \frac{20}{3}$ meters.

**b)** To find the TOTAL distance travelled use $\int_0^5 |v(t)|\,dt$. We need to know where $v(t)$ is positive and negative. Find the roots: $v(t) = t^2 - 6t + 8 = (t-2)(t-4)$. So $t = 2$ or $t = 4$. A check between the roots shows that $v(0) = 8$, $v(3) = -1$ and $v(5) = 3$. So we have to split the integral into three pieces and change the sign on the piece from $t = 2$ to 4.

$$\int_0^5 |v(t)|\,dt = \int_0^2 t^2-6t+8\,dt - \int_2^4 t^2-6t+8\,dt + \int_4^5 t^2-6t+8\,dt = \frac{t^3}{3}-3t^2+8t\Big|_0^2 - \left(\frac{t^3}{3} - 3t^2 + 8t\right)\Big|_2^4 + \left(\frac{t^3}{3} - 3t^2 + 8t\right)\Big|_4^5$$

$$= \frac{8}{3} - 12 + 16 - 0 + \left(\frac{64}{3} - 48 + 32 - \left[\frac{8}{3} - 12 + 16\right]\right) + \left(\frac{125}{3} - 75 + 40 - \left[\frac{64}{3} - 48 + 32\right]\right) = \frac{28}{3}\text{ m.}$$

**5. a)** Future value $= P(20) = P(0) + \int_0^{20} 30(1+\sqrt{t})\,dt = 250 + 30\left(t + \frac{2t^{3/2}}{3}\right)\Big|_0^{20} = 250 + 30\left(20 + \frac{2\cdot 20^{3/2}}{3} - 0\right) = 2639$ people.

**b)** Future pop $P(t) = P(0) + \int_0^t 30(1+\sqrt{x})\,dx = 250 + 30\left(x + \frac{2x^{3/2}}{3}\right)\Big|_0^t = 250 + 30\left(t + \frac{tx^{3/2}}{3}\right)$

**6. a)** $E(24) - E(0) = \int_0^{24} E'(t)\,dt = \int_0^{24} 300 - 200\sin\left(\frac{\pi t}{12}\right)\,dt = 300t + 2400\cos\left(\frac{\pi t}{12}\right)\Big|_0^{24} = (7200+2400) - (0+2400) =$ 7200MWh.

**b)** Midnight is $t = 6$, and 6 a.m is $t = 12$. So

$$E(12) - E(6) = \int_6^{12} 300 - 200\sin\left(\frac{\pi t}{12}\right)\,dt = 300t + 2400\cos\left(\frac{\pi t}{12}\right)\Big|_6^{12} = (3600-2400) - (1800+0) = 600 \text{ MWh}$$

It is less than one-fourth of the entire day's consumption.

**c)** Day: $\frac{7200\text{MWh}}{450\text{kWh/kg}} = 1600\text{kg}$ Year: 5,840,000kg.

**d)** Day: $\frac{7200\text{MWh}}{1600\text{kWh/g}} = 450\text{g}$ Year: 164,250g.

**7. a)** $\int \tan^2(\pi x)\,dx = \int \sec^2(\pi x) - 1\,dx = \frac{1}{\pi}\tan(\pi x) - x + c.$

**b)** $\int \tan(2\pi x)\,dx = \frac{1}{2\pi}\ln|\sec(2\pi x)| + c.$

**c)** $u = e^{2x^2}$, $du = 4xe^{2x^2}dx$. $\frac{1}{4}\int \sec u\,du = \frac{1}{4}\ln|\sec u + \tan u| + c = \frac{1}{4}\ln|\sec e^{2x^2} + \tan e^{2x^2}| + c.$

**8. a)** Mentally adjust: $-\frac{1}{8}\ln|\sec(-8t) + \tan(-8t)| + c$ or do by $u$-substitution.

**b)** $u = 2\sin x$, $\frac{1}{2}du = \cos x dx$. $\frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + c = \frac{1}{2}e^{2\sin x} + c.$

**c)** $u = 2\phi^4 + 3$, $\frac{1}{8}du = \phi^3 d\phi$. $\frac{1}{8}\int \sec^2(u)\,du = \frac{1}{8}\tan u + c = \frac{1}{8}\tan(2\phi^4+3) + c.$

**d)** $u = 3t+1$, $2du = 6dt$. $2\int u^{-1}\,du = 2\ln|u| + c = 2\ln|3t+1| + c.$

**e)** $u = \sqrt{x}$, $2du = \frac{1}{\sqrt{x}}dx$. $2\int \tan u\,du = 2\ln|\sec u| + c = 2\ln|\sec(\sqrt{x})| + c.$

**f)** $u = \tan 3x$, $\frac{1}{3}du = \sec^2 3x dx$. $\frac{1}{3}\int u^{-4}\,du = -\frac{1}{9}u^{-3} + c = -\frac{1}{9}(\tan 3x)^{-3} + c.$

**g)** $\int \frac{t}{\sqrt{4-16t^4}}\,dt = \frac{1}{2}\int \frac{t}{\sqrt{1-4t^4}}\,dt$. $u^2 = 4t^4$, so $u = 2t^2$, $du = 4t\,dt$, $\frac{1}{4}du = t\,dt$.

$$\frac{1}{2}\int \frac{t}{\sqrt{1-4t^4}}\,dt = \frac{1}{8}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{8}\arcsin u + c = \frac{1}{8}\arcsin 2t^2 + c$$

**9.** Similar integrals done differently.

**a)** Divide : $\int \frac{1+x^4}{x}\,dx = \int \frac{1}{x} + x^3\,dx = \ln|x| + \frac{1}{4}x^4 + c$

**b)** $u = 1 + x^2 \Rightarrow \frac{1}{2}du = xdx$ $\quad\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln(1+x^2) + c$

**c)** $u^2 = x^6 \Rightarrow u = x^3 \Rightarrow \frac{1}{3}du = x^2dx$ $\quad\int \frac{x^2}{1+x^6}\,dx = \frac{1}{3}\int \frac{1}{1+u^2}\,du = \frac{1}{3}\arctan u + c = \frac{1}{3}\arctan(x^3) + c$

**10. a)** Note the 'mental adjustment.' $\int \cos^2(8x)\,dx = \int \frac{1}{2} + \frac{1}{2}\cos(16x)\,dx = \frac{1}{2}x + \frac{1}{32}\sin(16x) + c.$

**b)** $\int \sin^2(\pi x)\,dx = \int \frac{1}{2} - \frac{1}{2}\cos(2\pi x)\,dx = \frac{1}{2}x + \frac{1}{4\pi}\sin(2\pi x) + c.$

**11. a)** Let $u = \sqrt[3]{x+1}$ so cubing gives $u^3 = x+1$. So $u^3 - 1 = x$ and $3u^2\,du = dx$ Wow! I like the way we get $dx$ here! Now we can substitute.

$$\int x\sqrt[3]{x+1}\,dx = \int (u^3-1)\cdot u\cdot 3u^2\,du = \int 3u^6 - 3u^3\,du = \tfrac{3}{7}u^7 - \tfrac{3}{4}u^4 + c = t\tfrac{3}{7}(x+1)^{7/3} - \tfrac{3}{4}(+1)^{4/3} + c. \text{ Slick!}$$

**b)** Let $u = \sqrt{x-1}$ so $u^2 = x - 1$. So $u^2 + 1 = x$ and $2u\,du = dx$. Now we can substitute.

$$\int \frac{x}{\sqrt{x-1}}\,dx = 2\int \frac{u^2+1}{u}\cdot 2u\,du = \int 2^2 + 2\,du = u^3 + 2u + c = (x-1)^{3/2} + 2\sqrt{x+1} + c. \quad \text{Same.}$$

**12.** **a)** $\int x^2 \cos^2(x^3)\sin(x^3)\,dx = \int \frac{1}{3}\cos^2 u \sin u\,du$. Let $w = \cos u$ and $dw = -\sin u\,du$.

$$\int \frac{1}{3}\cos^2 u \sin u\,du = \int -\frac{1}{3}w^2\,dw = -\frac{1}{9}w^3 + c = -\frac{1}{9}\cos^3 u + c = -\frac{1}{9}\cos^3(x^3) + c.$$

**b)** $u = \sqrt{1+x} \Rightarrow du = \dfrac{1}{2\sqrt{1+x}} \Rightarrow 2\sqrt{1+x}\,du = dx$ So $2u\,du = dx$. Whoa!

$$\int \frac{1}{\sqrt{1+\sqrt{1+x}}}\,dx = \int \frac{2u}{\sqrt{1+u}} \overset{\text{LikeProb3(c)}}{=} 2\left[\frac{2}{3}(u+1)^{3/2} + 2(u+1)^{1/2}\right] + c = \frac{4}{3}(1+\sqrt{1+x})^{3/2} - 4(1+\sqrt{1+x})^{1/2} + c.$$

**13.** Note: The initial velocity and position are $v_0 = 0$ and $s_0$ since the sprinter starts the race not moving at the start line.

**a)** part Ave acc $= \frac{1}{b-a}\int_a^b f(t)\,dt = \frac{1}{4-0}\int_0^4 6t^{1/2}\,dt = \frac{1}{4}\cdot 4t^{3/2}\Big|_0^4 = 1(8-0) = 8$ m/s$^2$.

**b)** Velocity is $v(t) = \int 6t^{1/2}\,dt = 4t^{3/2} + v_0 = 4t^{3/2}$. So

$$\text{Ave vel} = \frac{1}{4-0}\int_0^4 4t^{3/2}\,dt = \frac{1}{4}\cdot\frac{8}{5}t^{5/2}\Big|_0^4 = \frac{64}{5} \text{ m/s.}$$

**c)** Position is $s(t) = \int 4t^{3/2}\,dt = \frac{8}{5}t^{5/2} + s_0 = \frac{8}{5}t^{5/2}$.

$$\text{Ave pos} = \frac{1}{4-0}\int_0^4 \frac{8}{5}t^{5/2}\,dt = \frac{1}{4}\cdot\frac{16}{35}t^{7/2}\Big|_0^4 = \frac{512}{35} \text{ m.}$$

**14.** Note limits. From FTC $g(x) = \frac{d}{dx}\left[-\int_{-1}^{x} g(t)\,dt\right] = \frac{d}{dx}(-\arctan(3x^2)) = -\frac{6x}{1+9x^4}$. So $g(1) = -\frac{6}{10}$.