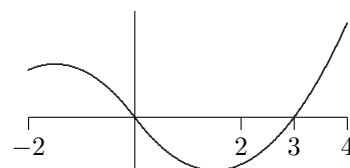
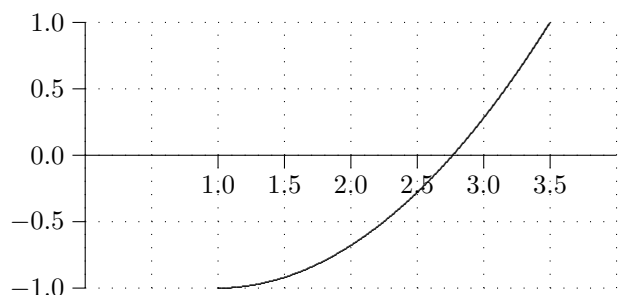


# Math 131 PracTest 1

0. Be sure to review Lab 4 (and all labs). There are lots of good questions on it.

1. a) State the Mean Value Theorem and draw a graph that illustrates.  
 b) Name an important theorem where the Mean Value Theorem was used in the proof.  
 c) In your own words, state the definition of the definite integral of a continuous function on a closed interval  $[a, b]$  as a limit of some process.  
 d) State the Fundamental Theorem of Calculus (FTC I).  
 e) Determine  $\frac{d}{dx} \left[ \int_{x^3}^1 \sin^2(t) dt \right]$ . What theorem(s) apply?  
 f) If  $\int_0^x g(t) dt = \frac{\sin x}{e^x}$ , what is  $g(0)$ ? Explain.

2. Draw and then estimate Right(5) for the graph of  $f$  on  $[1, 3.5]$  on the left below. Be careful of the scale.



3. In the graph of  $f$  on the right above, assume that  $\int_{-2}^4 f(x) dx = 1$ ,  $\int_{-2}^0 f(x) dx = 3$ , and  $\int_{-2}^3 f(x) dx = -1.2$ . Evaluate the following.

a)  $\int_3^4 f(x) dx$       b)  $\int_0^3 f(x) dx$       c)  $\int_{-1}^5 2f(x-1) dx$       d)  $\int_{-2}^0 f(x) - 4 dx$

e) If  $f(x)$  is symmetric about the origin (an ‘odd’ function), what is  $\int_0^2 f(x) dx$ ?

4. Calculate these “look-alike” indefinite integrals.

a)  $\int \sqrt{4t-1} dt$       b)  $\int \frac{1}{\sqrt{25-4t^2}} dt$       c)  $\int \frac{t}{\sqrt{1-4t^2}} dt$       d)  $\int t\sqrt{4-t} dt$       e)  $\int \frac{t}{1+4t^4} dt$   
 f)  $\int \frac{t^3}{1+4t^4} dt$       g)  $\int \frac{t^2}{9+4t^6} dt$       h)  $\int \frac{1+4t^6}{t^2} dt$       i)  $\int \frac{e^{2x}}{\sqrt{25-e^{4x}}} dx$       j)  $\int \frac{1}{3+12t^2} dt$

5. a) If an antiderivative of  $f(x)$  is  $\sin x - x \sin x + 1$ , what was  $f(x)$ ?  
 b) On a recent test Judy said that  $\int 2 \cos x \sin x dx = \sin^2 x + c$ . Elaine said that  $\int 2 \cos x \sin x dx = -\cos^2 x + c$ . I gave them both full-credit. How can both be correct if their answers are different? Did I make a mistake?

6. a) Fill in the table for Right( $n$ ) for  $\int_1^3 3(x^2 - 1) dx$ . Be sure to simplify  $f(x_i)$ .

$f(x)$	$[a, b]$	$\Delta x$	$x_i$	$f(x_i)$	Right( $n$ )

- b) If you were to graph  $f(x)$  on  $[1, 3]$  it would be an increasing function. Is Right( $n$ ) an over or under estimate of  $\int_1^3 3(x^2 - 1) dx$ ? Explain.  
 c) Find  $\int_1^3 3(x^2 - 1) dx$  using a limit of Riemann sums. Check your answer by evaluating  $\int_1^3 3(x^2 - 1) dx$  using the FTC.  
 d) Think: On a test once, I asked students to compute Right(100) and Right(200) for  $f(x) = e^x$  on an interval  $[a, b]$ . A student correctly computed both with his calculator but failed to label which is which. Which of her two values is Right(100): 47.6828 or 47.4455? **Explain** clearly how you can tell.

7. Determine these indefinite and definite integrals:

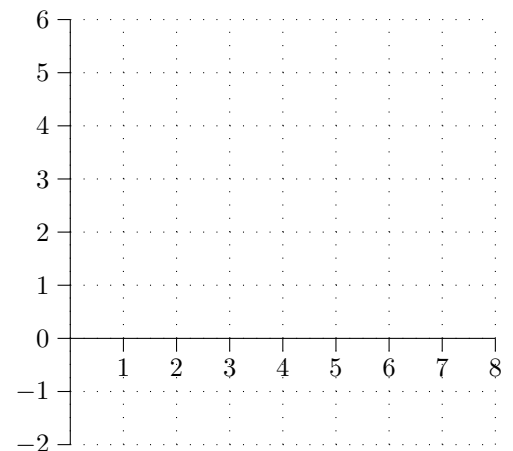
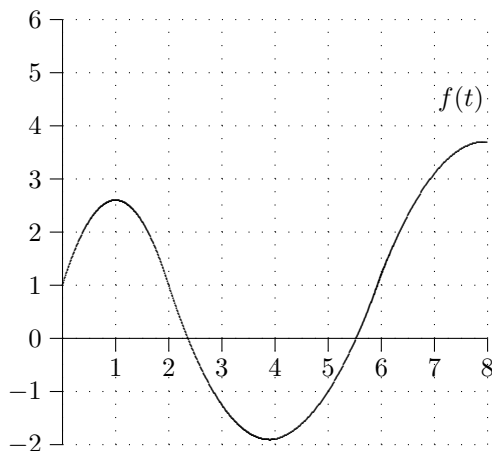
a)  $\int 8 \sec(4\pi x) \tan(4\pi x), dx$      
 b)  $\int \frac{e^{\sqrt{t}}}{\sqrt{t}} dt$      
 c)  $\int z \tan(6z^2 + 1) dz$      
 d)  $\int \sin(\sin t) \cos t dt$   
 e)  $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$      
 f)  $\int_{\pi}^{\pi} x^2 \sin^5(x) dx$      
 g)  $\int \sec\left(\frac{\pi}{4}x\right) dx$      
 h)  $\int \sec^2\left(\frac{\pi}{4}x\right) dx$

8. The graph of  $f(t)$  is given below on the left. Answer the following questions about  $F(x) = \int_0^x f(t) dt$ .

- On what interval(s) is  $F$  increasing? Explain.
- At what point(s), if any, does  $F$  have a local min? Explain.
- On what interval(s) is  $F$  concave down? Explain.
- Does  $F$  have any points of inflection? Explain.
- Fill in the the table below. Your values should be reasonably accurate.

$x$	0	1	2	3	4	5	6	7	8
$F(x) = \int_0^x f(t) dt$									

f) Graph  $F(x)$  on the axes on the right using your table values and knowledge about maxs and mins.



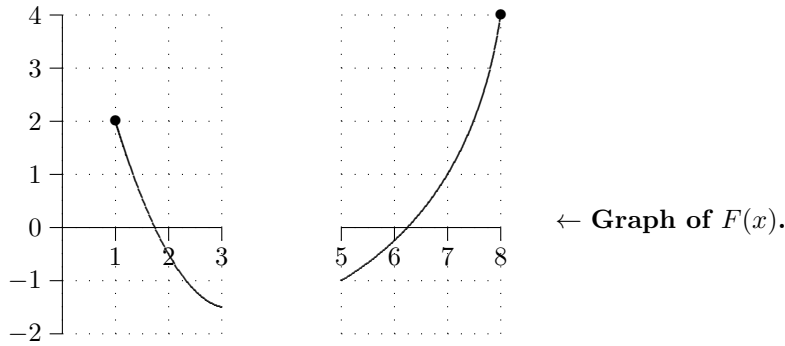
9. **Think and Interpret:** The **velocity** of an object  $v(t)$  in m/s is graphed above on the left (same graph as in previous question).

- When is the object moving backwards? Explain.
- Assume that the initial position of the object is  $s(0) = 0$ . Fill in the row in the table for the position  $s(t)$  for the object. Then graph the position function on the axes above on the right.

$t$	0	1	2	3	4	5	6	7	8
Position $s(t)$									
Distance Traveled = $\int_0^t  v(t)  dt$									

- What is  $v_{\text{ave}}$  on the interval  $[0, 8]$ ? (Hint: Use the  $v_{\text{ave}}$  formula and your work above.) At what time(s)  $t$  did  $v_{\text{ave}}$  occur (i.e., when was  $v(t)$  actually equal to  $v_{\text{ave}}$ ? Explain.
- Speed is  $|v(t)|$ . Graph the speed function for the object on the axes on the left. Distance traveled is the integral of the speed:  $\int_0^t |v(t)| dt$ . Fill in the row in the table above for the distance traveled by the object.

10. When making up this question, the printer jammed and only part of the graph of a differentiable function  $F(x)$  was printed out, as shown below. Nonetheless, the graph still provides enough information for you to precisely evaluate  $\int_1^8 F'(x) dx$ . What is the value of this integral. (Look carefully at the integrand.)



11. a) Suppose that the acceleration of an object is  $a(t) = e^{2t}$  and that  $v(0) = 2$  and  $s(0) = 1$ , where  $v$  and  $s$  have their usual meaning. Find  $s(t)$ .  
 b) Suppose instead that  $a(t) = 24t^2 + 12t$  and that  $s(0) = 4$  and  $s(1) = 8$ . Find  $s(t)$ .
12. Evaluate the following expression:

$$\frac{d}{dx} \int_{x^2}^2 4t^3 e^t dt$$

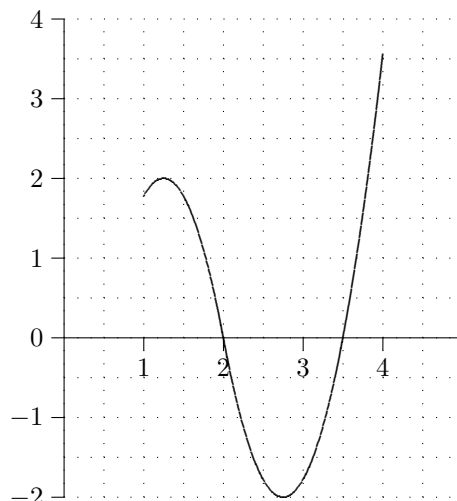
13. On Mars the acceleration due to gravity equals 0.39 that of earth or  $-12.5$  ft/sec/sec. Suppose that a Martian calculus student throws his calculus text *upward* at 25 ft/sec off the roof of a building.  
 a) If it takes 6 seconds to hit the ground, how high is the building?  
 b) What was the velocity of the book when it the ground?
14. Let  $f(x) = x^2 - x$  on the interval  $[1, 3]$ . Suppose we want to find  $\text{Right}(n)$  for this function.  
 a) Determine the expressions for  $\Delta x$ ,  $x_k$ , and  $f(x_k)$ . Be sure to simplify where possible.  
 b) Determine and simplify the expression for  $\text{Right}(n)$ .  
 c) Evaluate  $\int_1^3 x^2 - x dx$  as a limit of Riemann sums.  
 d) Check your answer by using the Fundamental Theorem of Calculus.

15. One of the integral properties states that if we switch the limits of integration, the sign of the integral is switched:

$$\int_b^a f(x) dx = - \int_a^b f(x) dx.$$

Give a brief explanation of why this should be so. Hint: Think about  $\Delta x$ .

16. Use the graph  $f$  below to draw and then estimate the **left-hand** sum  $\text{Left}(6)$  for the given function on  $[1, 4]$ .



17. Three similar integrals.

a)  $\int \frac{1+x^4}{x} dx$       b)  $\int \frac{x}{1+x^2} dx$       c)  $\int \frac{x^2}{1+x^6} dx$

18. My Honda Accord accelerates from 0 to 88 ft/sec (60 mph) in 13 seconds. Assume that acceleration is a constant,  $a$ .

- a) Find the velocity function of the car (using the velocities at the two times you can eliminate any constants from this function).  
 b) How far does it travel in this 13 second period?

19. A balloon, rising vertically with a velocity of 8 ft/s, releases a sandbag at the instant it is 64 ft above the ground.

- a) How many seconds after its release will the bag strike the ground? Remember  $a(t) = -32 \text{ ft/s}^2$ .  
 b) At what velocity does it hit the ground?

20. a) Find the average value of  $f(t) = \frac{t}{1+3t^2}$  on  $[0, 2]$ .

b) Find the average value of  $f(t) = 2t^{999} + \pi t^{1001} - 6t^{47} - 17t^5 + \cos(\pi t)$  on  $[-1/2, 1/2]$ . Why is this EZ?

21. a) Page 417 #9. (Answers in text.)

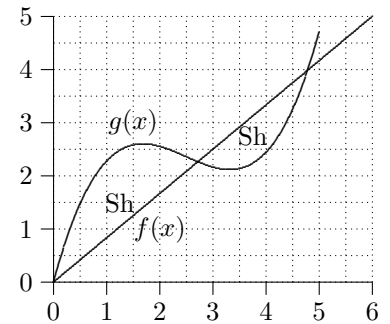
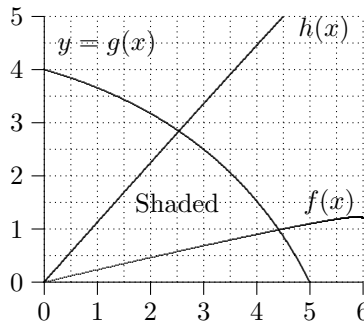
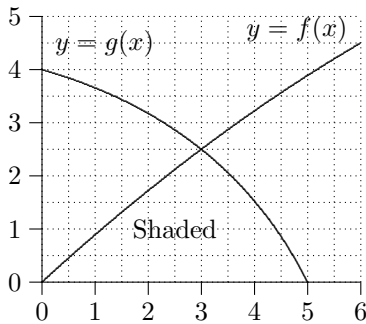
b) Page 417 #23. (Answers in text.)

22. Area between curves: See Assignment from Day 12 and the associated WeBWork problems (and all of the examples on line in the Notes for Day 12).

23. Determine

a)  $\int \cos^2(6x) dx$       b)  $\int \sin^2(-2\pi x) dx$

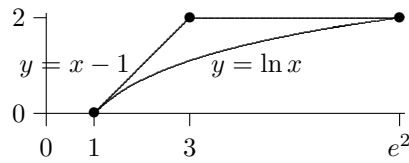
24. Set up the integrals using the functions  $f(x)$ ,  $g(x)$ , and  $h(x)$  and their points of intersection that would be used to find the shaded areas in the three regions below.



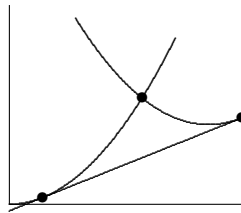
25. Sketch the regions for each of the following problems before finding the areas.

- a) (Do Later) Find the area enclosed by the curves  $y = x^3$  and  $y = x^2$ . (Ans:  $1/12$ )  
 b) Find the area enclosed by the curves  $y = x^3 + x$  and  $y = 3x^2 - x$ . (Ans:  $1/2$ )  
 c) Find the area enclosed by  $x = y^2 + 1$  and  $x = 2y + 9$ . Integrate along the  $y$ -axis. (Ans: 36)  
 d) Find the area in the first quadrant enclosed by the curves  $y = \sqrt{x-1}$ ,  $y = 3-x$ , the  $x$ -axis, and the  $y$ -axis by using definite integrals along the  $y$ -axis. (Ans:  $10/3$ , if you get  $9/2$ , you have the wrong region.)  
 e) Find the area of the wedge-shaped region below the curves  $y = \sqrt{x-1}$ ,  $y = 3-x$ , and above the  $x$ -axis. Integrate along either axis: your choice! (Note: Not the same as (d). Ans:  $7/6$ .)  
 f) The curves  $2\sin x$  and  $\tan x$  intersect at  $x = 0$ . Find  $b$ , where  $b$  is the smallest **positive** value of  $x$  where the curves meet. (Hint: Write  $\tan x$  in terms of sines and cosines.) Finally, determine the area between  $2\sin x$  and  $\tan x$  on the interval  $[0, b]$ . (Ans:  $1 - \ln 2$ )  
 g) Find the area between the curves  $f(x) = \cos x + \sin x$  and  $g(x) = \cos x - \sin x$  over  $[0, 2\pi]$ . (Ans: 8)

26. a) Sketch the region (use your calculator?) and find the area under  $y = \arcsin x$  on the interval  $[0, 1]$ . Hint: switch axes. (Ans:  $\pi/2 - 1$ )
- b) **\*From a test last year:** Consider the region bounded by  $y = \ln x$ ,  $y = 2$ , and  $y = x - 1$  shown below. Find the area of this region. Hint: Turn your head! (Ans:  $e^2 - 5$ )



27. Find the region enclosed by the three curves  $y = x^2$ ,  $y = x^2 - 12x + 48$ , and  $y = 2x - 1$ . You will need to find the intersections. (Ans: 18)



28. Find the area of the region in the first quadrant enclosed by  $y = 9 - x$ ,  $y = x\sqrt{x+1}$ , and the  $y$ -axis. Hint: The two curves meet at the point  $(3, 6)$ .

# Math 131: Answers to Practest 1

1. a) Mean Value Theorem: See page 292 in your text.
- b) FTC II or Mean Value Theorem for Integrals.
- c) Partition the interval  $[a, b]$  into  $n$  equal width subintervals using points

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b.$$

The definite integral of a continuous function  $f$  on a closed interval  $[a, b]$  is

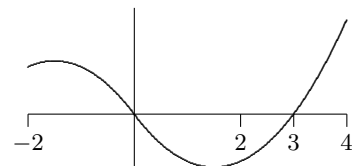
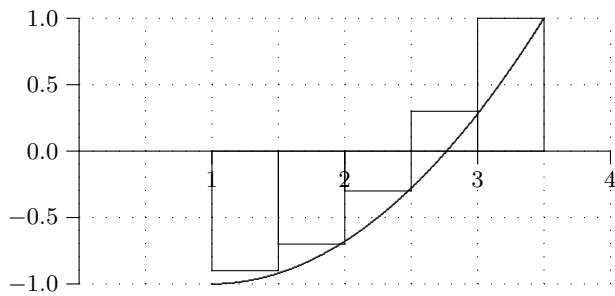
$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x,$$

where  $\Delta x = \frac{b-a}{n}$  and  $c_i$  is some point in the  $i$ -th subinterval.

d)  $\frac{d}{dx} \left[ \int_{x^3}^1 \sin^2(t) dt \right] = \frac{d}{dx} \left[ - \int_1^{x^3} \sin^2(t) dt \right] = \sin^2(x^3) \cdot 3x^2$ , where we used FTC II and the chain rule.

e) FTC II implies  $g(x) = \frac{d}{dx} \left[ \int_0^x g(t) dt \right] = \frac{d}{dx} \left[ \frac{\sin x}{e^x} \right] = \frac{e^x \cos x - e^x \sin x}{e^{2x}}$ . So  $g(0) = \frac{e^0 \cos 0 - e^0 \sin 0}{e^0} = 1$ .

2.  $\Delta x = \frac{3.5-1}{5} = \frac{1}{2}$ . So  $\text{Right}(5) = (-.9) \cdot \frac{1}{2} + (-0.7) \cdot \frac{1}{2} + (-0.3) \cdot \frac{1}{2} + (0.3) \cdot \frac{1}{2} + (1) \cdot \frac{1}{2} = -0.3$ .



3. Use basic integral properties.

a)  $\int_3^4 f(x) dx = \int_{-2}^4 f(x) dx - \int_{-2}^3 f(x) dx = 1 - (-1.2) = 2.2$

b)  $\int_0^3 f(x) dx = \int_{-2}^3 f(x) dx - \int_{-2}^0 f(x) dx = -1.2 - 3 = -4.2$

d)  $\int_{-2}^0 f(x) - 4 dx = 3 + (-4)[0 - (-2)] = -5$

c)  $\int_{-1}^5 2f(x-1) dx = 2 \int_{-2}^4 f(x) dx = 2$  (horizontal shift)

e)  $\int_0^2 f(x) dx = - \int_{-2}^0 f(x) dx = -3$

4. a)  $u = 4t - 1, \frac{1}{4} du = dt. \frac{1}{4} \int u^{1/2} du = \frac{1}{6} u^{3/2} + c = \frac{1}{6} (4t - 1)^{3/2} + c.$

b) Use  $\int \frac{a^2}{\sqrt{1-u^2}} du = \arcsin \frac{u}{a} + c$  with  $a^2 = 25$  so  $a = 5, u^2 = 4t^2$ , so  $u = 2t, \frac{1}{2} du = dt. \frac{1}{2} \int \frac{1}{\sqrt{5^2-u^2}} du = \frac{1}{2} \arcsin(\frac{u}{5}) + c = \frac{1}{2} \arcsin(\frac{2t}{5}) + c.$

c)  $u = 1 - 4t^2, du = -8t dt, -\frac{1}{8} du = t dt. \int \frac{t}{\sqrt{1-4t^2}} dt = -\frac{1}{8} \int \frac{1}{u^{1/2}} du = -\frac{1}{4} u^{1/2} + c = -\frac{1}{4} \sqrt{1-4t^2} + c.$

d) Method 1:  $u = \sqrt{4-t},$  so  $u^2 = 4-t,$  so  $u^2 - 4 = -t$  so  $2u du = dt. \int t\sqrt{4-t} dt = \int (u^2 - 4) \cdot u \cdot 2u du = \int 2u^4 - 8u^2 du = \frac{2}{5} u^5 - \frac{8}{3} u^3 + c = \frac{2}{5} u^5 - \frac{8}{3} u^3 + c = \frac{2}{5} (4-t)^{5/2} - \frac{8}{3} (4-t)^{3/2} + c$

d) Method 2:  $u = 4-t,$  so  $4-u = t$  and  $-du = dt. - \int (4-u)u^{1/2} du = - \int 4u^{1/2} - u^{3/2} du = -\frac{8}{3} u^{3/2} + \frac{2}{5} u^{5/2} + c = \frac{2}{5} (4-t)^{5/2} - \frac{8}{3} (4-t)^{3/2} + c.$

e)  $u^2 = 4t^4, u = 2t^2, \frac{1}{4} du = t dt. \frac{1}{4} \int \frac{1}{1+u^2} du = \frac{1}{4} \arctan u + c = \frac{1}{4} \arctan(2t^2) + c.$

f)  $u = 1 + 4t^4, \frac{1}{16} du = t^3 dt. \frac{1}{16} \int \frac{1}{u} du = \frac{1}{16} \ln |u| + c = \frac{1}{16} \ln(1 + 4t^4) + c.$

g)  $a^2 = 9 = 3^2, u^2 = 4t^6, u = 2t^3, \frac{1}{6} du = t^2 dt. \frac{1}{6} \int \frac{1}{3^2+u^2} du = \frac{1}{6} \cdot \frac{1}{3} \arctan(\frac{u}{3}) + c = \frac{1}{18} \arctan(\frac{2t^3}{3}) + c.$

h)  $\int \frac{1+4t^6}{t^2} dt = \int t^{-2} + 4t^4 dt = -t^{-1} + \frac{4}{5} t^5 + c. (Just divide first by t^2.)$

i) Use  $\int \frac{a^2}{\sqrt{1-u^2}} du = \arcsin \frac{u}{a} + c$  with  $a^2 = 25$  so  $a = 5, u^2 = e^{4x},$  so  $u = e^{2x}, \frac{1}{2} du = e^{2x} dx. \frac{1}{2} \int \frac{1}{\sqrt{5^2-u^2}} du = \frac{1}{2} \arcsin(\frac{u}{5}) + c = \frac{1}{2} \arcsin(\frac{e^{2x}}{5}) + c.$

j)  $u^2 = 4t^2 \Rightarrow u = 2t \Rightarrow du = 2 dt \Rightarrow \frac{1}{2} du = dt$ . So

$$\int \frac{1}{\sqrt{1-4t^2}} dt = \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \arcsin u + c = \frac{1}{2} \arcsin 2t + c.$$

k)  $\int \frac{1}{3+12t^2} dt = \frac{1}{3} \int \frac{1}{1+4t^2} dt$ . Use  $u^2 = 4t^2 \Rightarrow u = 2t \Rightarrow du = 2 dt \Rightarrow \frac{1}{2} du = dt$ . So

$$\frac{1}{3} \int \frac{1}{1+4t^2} dt = \frac{1}{3} \cdot \frac{1}{2} \int \frac{1}{1+u^2} du = \frac{1}{6} \arctan u + c = \frac{1}{6} \arctan 2t + c.$$

5. a) Since  $\int f(x) dx = \sin x - x \sin x + 1$ , then  $f(x) = \frac{d}{dx}(\sin x - x \sin x + 1) = \cos x - \sin x - x \cos x$ .

b) No. Just take the derivatives of each:  $\frac{d}{dx}(\sin^2 x + c) = 2 \sin x \cos x$  while  $\frac{d}{dx}(-\cos^2 x + c) = -2 \cos x(-\sin x) = 2 \sin x \cos x$  also. The two antiderivatives differ by a constant since  $\sin^2 x = -\cos^2 x + 1$ .

6. a)

$f(x)$	$[a, b]$	$\Delta x$	$x_i$	$f(x_i)$	Right( $n$ )
$3(x^2 - 1)$	$[1, 3]$	$\frac{2}{n}$	$1 + \frac{2i}{n}$	$3\left(\frac{4i^2}{n^2} + \frac{4i}{n}\right)$	$\sum_{i=1}^n 3\left(\frac{4i^2}{n^2} + \frac{4i}{n}\right) \left(\frac{2}{n}\right)$

b) Since the function is increasing ( $f'(x) = 6x > 0$  on  $[1, 3]$ ), Right( $n$ ) is an overestimate of  $\int_1^3 3x^2 - 3 dx$  because  $f(x_i)$  will be greater than any value of  $f(x)$  in  $[x_{i-1}, x_i]$

c) Right( $n$ ) =  $3 \sum_{i=1}^n \left[ \frac{4i^2}{n^2} + \frac{4i}{n} \right] \cdot \frac{2}{n} = \frac{24}{n^3} \sum_{i=1}^n i^2 + \frac{24}{n^2} \sum_{i=1}^n i = \frac{24}{n^3} \left[ \frac{n(n+1)(2n+1)}{6} \right] + \frac{24}{n^2} \left[ \frac{n(n+1)}{2} \right]$

$$= 4 \left[ \frac{2n^2 + 3n + 1}{n^2} \right] + \frac{12n + 12}{n} = 8 + \frac{12}{n} + \frac{4}{n^2} + 12 + \frac{12}{n} = 20 + \frac{24}{n} + \frac{4}{n^2}$$

So  $\int_1^3 3(x^2 - 1) dx = \lim_{n \rightarrow \infty} \text{Right}(n) = 20$  and using the FTC,  $\int_1^3 3x^2 - 3 dx = x^3 - 3x \Big|_1^3 = (27 - 9) - (1 - 3) = 20$ .

d) Since  $e^x$  is an increasing function Right( $n$ ) is an overestimate. Since the estimates improve (smaller overestimate) as  $n$  gets larger, then Right(100) > Right(200). So Right(100) = 47.6828.

7. a)  $\frac{8}{4\pi} \sec(4\pi x) + c = \frac{2}{\pi} \sec(4\pi x) + c$ . (Mental adjustment)

b)  $u = \sqrt{t}$ ,  $2 du = \frac{1}{\sqrt{t}} dt$ .  $2 \int e^u du = 2e^u + c = 2e^{\sqrt{t}} + c$ .

c)  $u = 6z^2 + 1$ ,  $du = 12z dz$ ,  $\frac{1}{12} du = z dz$ . So  $\int z \tan(6z^2 + 1) dz = \frac{1}{12} \int \tan u du = \frac{1}{12} \ln |\sec u| + c = \frac{1}{12} \ln |\sec(6z^2 + 1)| + c$ .

d)  $u = \sin t$ ,  $du = \cos t dt$ .  $\int \sin u du = -\cos(u) + c = -\cos(\sin t) + c$ .

e)  $u = e^x + e^{-x}$ ,  $du = (e^x - e^{-x})dx$ . Change the limits: When  $x = 0$ ,  $u = e^0 + e^0 = 2$ ; when  $x = 1$ ,  $u = e + e^{-1}$ . So

$$\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \int_2^{e+e^{-1}} \frac{1}{u} du = \ln |u| \Big|_2^{e+e^{-1}} = \ln |e + e^{-1}| - \ln 2.$$

f)  $\int_{\pi^{\leftarrow}}^{\pi^{\leftarrow}} x^2 \sin^5(x) dx = 0$ . Integral property.

g)  $\int \sec\left(\frac{\pi}{4}x\right) dx = \frac{4}{\pi} \ln |\sec\left(\frac{\pi}{4}x\right) + \tan \sec\left(\frac{\pi}{4}x\right)| + c$ . "Adjust!"

h)  $\int \sec^2\left(\frac{\pi}{4}x\right) dx = \frac{4}{\pi} \tan\left(\frac{\pi}{4}x\right) + c$ . "Adjust!"

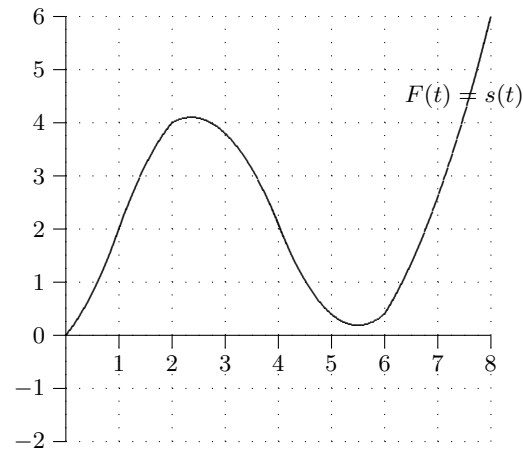
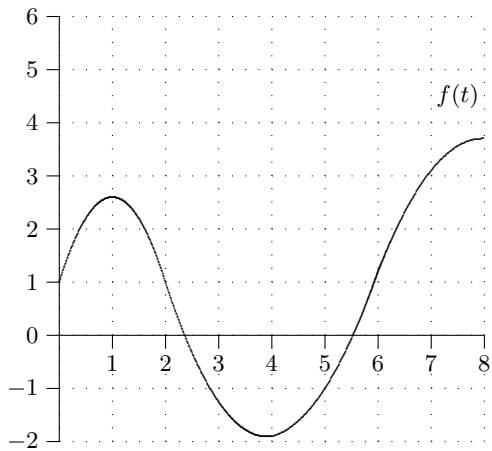
8. The graph of  $f(x)$  is given below on the left. Answer the following questions about  $F(x) = \int f(x) dx$ .

a)  $F$  increasing, so  $F' = f > 0$ :  $[0, 2.5]$  and  $[5.5, 8]$ .

b) Local min when  $F' = f$  changes from negative to positive:  $x = 5.5$

c)  $\int_0^b f(t) dt$  is just the net area under the curve from 0 to  $b$ .

$b$	0	1	2	3	4	5	6	7	8
$F(b) = \int_0^b f(t) dt$	0	2	4	3.8	2.1	0.4	0.4	2.6	6



9. a) When is the object moving backwards?  $v < 0$  so  $[2, 5, 5.5]$ .  
 b) Negative acceleration when  $a = v' < 0$ : so  $[1, 4]$ .  
 c) Position is the net area under the curve (same table as above).

$t$	0	1	2	3	4	5	6	7	8
Position $s(t)$	0	2	4	3.8	2.1	0.4	0.4	2.6	6
Distance Traveled	0	2	4	4.8	6.5	8.2	9	11.2	14.6

d)  $v_{\text{ave}} = \frac{1}{8-0} \int_0^8 v(t) dt = \frac{1}{8} [s(8) - s(0)] = \frac{1}{8} (6 - 0) = \frac{3}{4}$ .  $v_{\text{ave}}$  occurred at  $c = 2.2$  and  $5.8$ . [Use the graph of  $v(t) = f(t)$ .]

10. From the graph we see that  $F(1) = 2$  and  $F(8) = 4$ . So  $\int_1^8 F'(x) dx = F(x)|_1^8 = F(8) - F(1) = 4 - 2 = 2$ .

11. a) Integrate twice:  $v(t) = \int e^{2t} dt = \frac{1}{2}e^{2t} + c$ .  $v(0) = 2 = \frac{1}{2} + c \Rightarrow c = \frac{3}{2}$ .  $s(t) = \int \frac{1}{2}e^{2t} + \frac{3}{2} dt = \frac{1}{4}e^{2t} + \frac{3}{2}t + c$ .  
 $s(0) = 1 = \frac{1}{4} + c \Rightarrow c = \frac{3}{4}$ . So  $s(t) = \frac{1}{4}e^{2t} + \frac{3}{2}t + \frac{3}{4}$ .

b)  $v(t) = \int 24t^2 + 12t dt = 8t^3 + 6t^2 + c$ . So  $s(t) = \int 8t^3 + 6t^2 + c dt = 2t^4 + 2t^3 + ct + d$ . Now  $s(0) = 4 = d$ . So  $s(1) = 8 = 2 + 2 + c + 4 \Rightarrow c = 0$ . So  $s(t) = 2t^4 + 2t^3 + 4$ .

12. By FTC II  $\frac{d}{dx} \int_{x^2}^2 4t^3 e^t dt = \frac{d}{dx} \left( - \int_2^{x^2} 4t^3 e^t dt \right) = -4(x^2)^3 e^{x^2} \cdot 2x = -8x^7 e^{x^2}$ .

13. a) Given  $s''(t) = -12.5$ ,  $s'(0) = 25$ . Find  $s(0)$ . Then  $s'(t) = \int -12.5 dt = -12.5t + c$ . But  $s'(0) = 25 = -12.5(0) + c$  means  $c = 25$  and  $s'(t) = -12.5t + 25$ . Next,  $s(t) = \int -12.5t + 25 dt = -6.25t^2 + 25t + d$ . Now set  $s(6) = 0 = -6.25(6)^2 + 25(6) + d \rightarrow d = 75$  ft. So  $s(t) = -6.25t^2 + 25t + 75$ . And  $s(0) = 75$ .

b) Find  $s'(6) = -12.5(6) + 25 = -50$  ft/s.

14. Let  $f(x) = x^2 - x$  on the interval  $[1, 3]$ . Suppose we want to find  $\text{Right}(n)$  for this function.

a)  $\Delta x = \frac{2}{n}$ ,  $x_i = 1 + \frac{2i}{n}$ , and  $f(x_i) = (1 + \frac{2i}{n})^2 - (1 + \frac{2i}{n}) = 1 + \frac{4i}{n} + \frac{4i^2}{n^2} - (1 + \frac{2i}{n}) = \frac{2i}{n} + \frac{4i^2}{n^2}$ . Be sure to simplify where possible.

b) So  $\text{Right}(n) = \sum_{i=1}^n 3 \left( \frac{4i^2}{n^2} + \frac{2i}{n} \right) \left( \frac{2}{n} \right)$ .

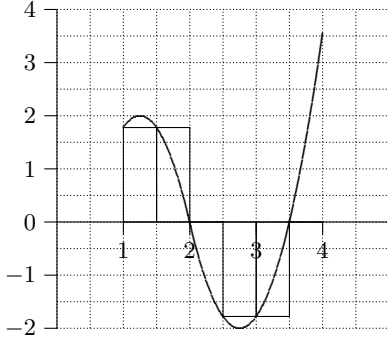
c)  $\text{Right}(n) = \sum_{i=1}^n \left[ \frac{4i^2}{n^2} + \frac{2i}{n} \right] \cdot \frac{2}{n} = \frac{8}{n^3} \sum_{i=1}^n i^2 + \frac{4}{n^2} \sum_{i=1}^n i = \frac{8}{n^3} \left[ \frac{n(n+1)(2n+1)}{6} \right] + \frac{4}{n^2} \left[ \frac{n(n+1)}{2} \right]$   
 $= \frac{4}{3} \left[ \frac{2n^2 + 3n + 1}{n^2} \right] + \frac{2n+2}{n} = \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} + 2 + \frac{2}{n} = \frac{14}{3} + \frac{6}{n} + \frac{4}{3n^2}$

So  $\int_1^3 x^2 - x dx = \lim_{n \rightarrow \infty} \text{Right}(n) = \frac{14}{3}$ .

d) By FTC,  $\int_1^3 x^2 - x dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 \Big|_1^3 = (9 - \frac{9}{2}) - (\frac{1}{3} - \frac{1}{2}) = \frac{14}{3}$ .



15. For  $\int_b^a f(x) dx$ , in the Riemann sum  $\Delta x = \frac{a-b}{n}$  which is the negative of the  $\Delta x$  for the Riemann sum for  $\int_a^b f(x) dx$  while the values of  $f(x_k)$  remain the same. Thus the sums will be the negatives of each other.



16.

Note  $\Delta x = \frac{4-1}{6} = \frac{1}{2}$ . So

$$L_6 = (1.8)\frac{1}{2} + (1.8)\frac{1}{2} + (0)\frac{1}{2} + (-1.8)\frac{1}{2} + (-1.8)\frac{1}{2} + (0)\frac{1}{2} = 0.$$

17. Similar integrals done differently and one extra.

a)  $\int \frac{1+x^4}{x} dx = \int \frac{1}{x} + x^3 dx = \ln|x| + \frac{1}{4}x^4 + c$

b)  $u = 1+x^2 \Rightarrow \frac{1}{2}du = x dx \quad \int \frac{x}{1+x^2} dx = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + c = \frac{1}{2} \ln(1+x^2) + c$

c)  $u^2 = x^6 \Rightarrow u = x^3 \Rightarrow \frac{1}{3}du = x^2 dx \quad \int \frac{x^2}{1+x^6} dx = \frac{1}{3} \int \frac{1}{1+u^2} du = \frac{1}{3} \arctan u + c = \frac{1}{3} \arctan(x^3) + c$

18. Given:  $a(t) = a$  constant.  $v_0 = 0$  ft/s.  $s_0 = 0$ . And  $v(13) = 88$  ft/s.

a) For constant acceleration:  $v(t) = at + v_0 = at$ . But  $v(13) = a \cdot 13 = 88 \Rightarrow a = \frac{88}{13}$ . So  $v(t) = \frac{88}{13}t$ .

b) For constant acceleration:  $s(t) = \frac{1}{2}at^2 + v_0t + s_0 = \frac{44}{13}t^2$ . Consequently,  $s(13) = \frac{44}{13}(13)^2 = 572$  ft.

19. Given:  $a(t) = -32$  ft/s<sup>2</sup> is constant.  $v_0 = 8$  ft/s.  $s_0 = 64$  ft. So check that

- $v(t) = -32t + 8$  ft/s

- and  $s(t) = -16t^2 + 8t + 64$  ft.

a) The bag hits the ground when the position  $s(t) = -16t^2 + 8t + 64 = 0$ . Using the quadratic formula the solution is  $t = \frac{1+\sqrt{65}}{4}$  (not  $\frac{1-\sqrt{65}}{4}$ ).

b) The velocity when it hits the ground is  $v(\frac{1+\sqrt{65}}{4}) = -32(\frac{1+\sqrt{65}}{4}) + 8 = -8\sqrt{65} \approx 64.5$  ft/s.

20. a)  $f_{ave} = \frac{1}{2-0} \int_0^2 \frac{t}{1+3t^2} dt$ .  $u = 1+3t^2$ ,  $du = 6t dt$ . When  $x = 0$ ,  $u = 1$ ;  $x = 2$ ,  $u = 13$ . So

$$f_{ave} = \frac{1}{2} \cdot \frac{1}{6} \int_1^{13} \frac{1}{u} du = \frac{1}{12} \ln|u| \Big|_1^{13} = \frac{\ln 13}{12}.$$

b) Use symmetry, split into odd and even terms:  $f_{ave} = \frac{1}{1/2 - (-1/2)} \int_{-1/2}^{1/2} 2t^{999} + \pi t^{1001} - 6t^{47} - 17t^5 + \cos(\pi t) dt$

$$= 1 \int_{-1/2}^{1/2} 2t^{999} + \pi t^{1001} - 6t^{47} - 17t^5 dt + 1 \int_{-1/2}^{1/2} \cos(\pi t) dt = 0 + 2 \int_0^2 \cos(\pi t) dt = \frac{1}{\pi} \sin \pi t \Big|_0^{1/2} = \frac{1}{\pi} (1 - 0) = \frac{1}{\pi}.$$

23. a) Note the 'mental adjustment.'  $\int \cos^2(6x) dx = \int \frac{1}{2} + \frac{1}{2} \cos(12x) dx = \frac{1}{2}x + \frac{1}{24} \sin(12x) + c$ .

b)  $\int \sin^2(-2\pi x) dx = \int \frac{1}{2} - \frac{1}{2} \cos(-4\pi x) dx = \frac{1}{2}x - \frac{1}{8\pi} \sin(4\pi x) + c$ .

24. Left:  $\int_0^3 f(x) dx + \int_3^5 g(x) dx$ . Middle:  $\int_0^{2.5} h(x) - f(x) dx + \int_{2.5}^{4.5} g(x) - f(x) dx$ . Right:  $\int_0^{2.7} g(x) - f(x) dx + \int_{2.7}^{4.7} f(x) - g(x) dx$ .

25. Sketch the curves! I have only done the integrals here.

a)  $A = \int_0^1 x^2 - x^3 dx = \frac{1}{3}x^3 - \frac{1}{4}x^4 \Big|_0^1 = (\frac{1}{3} - \frac{1}{4}) - 0 = \frac{1}{12}$ .

b)  $A_1 = \int_0^1 (x^3 + x) - (3x^2 - x) dx = \int_0^1 x^3 - 3x^2 + 2x dx = \frac{1}{4}x^4 - x^3 + x^2 \Big|_0^1 = (\frac{1}{4} - 1 + 1) - 0 = \frac{1}{4}$ .  $A_2 = \int_1^2 (3x^2 - x) - (x^3 + x) dx = -\frac{1}{4}x^4 + x^3 - x^2 \Big|_1^2 = (-4 + 8 - 4) - (-\frac{1}{4} + 1 - 1) = \frac{1}{4}$ .  $A_1 + A_2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ .

c)  $A = \int_{-2}^4 2y + 9 - (y^2 + 1) dy = \int_{-2}^4 2y + 8 - y^2 dy = y^2 + 8y - \frac{1}{3}y^3 \Big|_{-2}^4 = (16 + 32 - \frac{64}{3}) - (4 - 16 + \frac{8}{3}) = 36$ .

d) The region is bounded in part by the  $x$  and  $y$  axes in the first quadrant.  $A_1 = \int_0^1 y^2 + 1 dy = \frac{1}{3}y^3 + y \Big|_0^1 = (\frac{1}{3} + 1) - (0) = \frac{4}{3}$ .  $A_2 = \int_1^3 3 - y dy = 3y - \frac{1}{2}y^2 \Big|_1^3 = (9 - \frac{9}{2}) - (3 - \frac{1}{2}) = 2$ .  $A_1 + A_2 = \frac{10}{3}$ .

e) This is a wedge-shaped region under the curves  $y = 3 - x$ ,  $y = \sqrt{x - 1}$ , and above the  $x$ -axis. It can be done as a single region along the  $y$ -axes. The two curves meet at the point  $x = 2$  and  $y = 1$ .

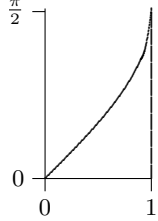
$$A = \int_0^1 (3 - y) - (y^2 + 1) dy = \int_0^1 2 - y - y^2 dy = 2y - \frac{1}{2}y^2 - \frac{1}{3}y^3 \Big|_0^1 = (2 - \frac{1}{2} - \frac{1}{3}) - (0) = \frac{7}{6}$$

f) When  $x \neq 0$ ,  $2 \sin x = \tan x \iff 2 \sin x = \frac{\sin x}{\cos x} \iff 2 \cos x = 1 \iff \cos x = \frac{1}{2} \iff x = \frac{\pi}{3}$ . On the interval from  $[0, \frac{\pi}{3}]$  which curve is on top? Plug in  $\frac{\pi}{4}$ :  $2 \sin(\frac{\pi}{4}) = \sqrt{2}$  while  $\tan(\frac{\pi}{4}) = 1$ , so  $\sin x$  is on top.

$$\int_0^{\frac{\pi}{3}} 2 \sin x - \tan x dx = -2 \cos x + \ln |\cos x| \Big|_0^{\frac{\pi}{3}} = \left(-1 + \ln \frac{1}{2}\right) - (-2 + \ln 1) = 1 - \ln 2$$

g)  $A_1 = \int_0^{\pi} (\cos x + \sin x) - (\cos x - \sin x) dx = \int_0^{\pi} 2 \sin x dx = -2 \cos x \Big|_0^{\pi} = 2 - (-2) = 4$ .  $A_2 = \int_{\pi}^{2\pi} (\cos x - \sin x) - (\cos x + \sin x) dx = \int_{\pi}^{2\pi} -2 \sin x dx = 2 \cos x \Big|_{\pi}^{2\pi} = 2 - (-2) = 4$ . Total area is 8.

26. a)



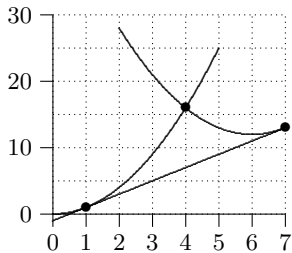
Integrate along the  $y$  axis.  $y = \arcsin x \Rightarrow x = \sin y$ .  
Be careful: The vertical line  $x = 1$  is now "on top."

$$A = \int_0^{\pi/2} 1 - \sin y dy = y + \cos y \Big|_0^{\pi/2} = \frac{\pi}{2} - 1$$

b) Again switch axes:  $y = \ln x \Rightarrow x = e^y$  and  $y = x - 1 \Rightarrow x = y + 1$ . So

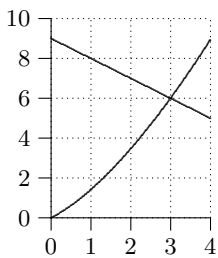
$$A = \int_0^2 e^y - (y + 1) dy = e^y - \frac{1}{2}y^2 - y \Big|_0^2 = (e^2 - 2 - 2) - 1 = e^2 - 5$$

27.



$$\begin{aligned} A &= \int_1^4 x^2 - (2x - 1) dx + \int_4^7 x^2 - 12x + 48 - (2x - 1) dx \\ &= \left[\frac{1}{3}x^3 - x^2 + x\right]_1^4 + \left[\frac{1}{3}x^3 - 7x^2 + 49x\right]_4^7 \\ &= \left(\frac{64}{3} - 16 + 4\right) - \left(\frac{1}{3} - 1 + 1\right) + \left(\frac{343}{3} - 343 + 343\right) - \left(\frac{64}{3} - 112 + 196\right) \\ &= 9 + 9 = 18. \end{aligned}$$

28.



$$A = \int_0^3 9 - x - x\sqrt{x+1} dx = \int_0^3 9 - x dx - \int_0^3 x\sqrt{x+1} dx$$

$$\text{But } \int_0^3 9 - x dx = 9x - \frac{x^2}{2} \Big|_0^3 = 27 - \frac{9}{2} - 0 = \frac{45}{2}$$

For the second integral,  $u = x + 1 \Rightarrow du = dx$  and  $u - 1 = x$ .

Limits:  $x = 0 \Rightarrow u = 1$ ,  $x = 3 \Rightarrow u = 4$ . So we get

$$\int_0^3 x\sqrt{x+1} dx = \int_1^4 (u - 1)u^{1/2} du = \int_1^4 u^{3/2} - u^{1/2} du$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} \Big|_1^4 = \frac{64}{5} - \frac{16}{3} - \left[\frac{2}{5} - \frac{2}{3}\right] = \frac{116}{15}$$

$$\text{Total Area: } \frac{45}{2} - \frac{116}{15} = \frac{443}{30}$$