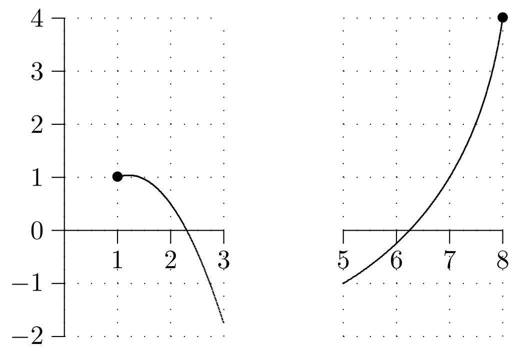
Math 131 Final Review

Extra Credit: Be the first to find typos in the questions or answers.

1. Do the series questions on Lab 14 for additional series problems beyond those included below.

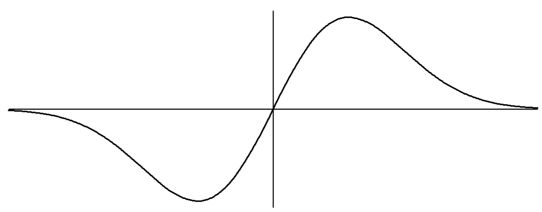
2. When making up this test, the printer jammed and only part of the graph of a differentiable function $F(x)$ was printed out, as shown below. Nonetheless, the graph still provides enough information for you to precisely evaluate $\int_1^8 F'(x)\,dx$. What is the value of this integral. (Look carefully at the integrand.)



← **Graph of $F(x)$.**

3. Let f be the function whose graph is given below. Use the information in the table, properties of the integral, and the **shape** of f to evaluate the given integrals.

a) $\displaystyle\int_3^0 f(x)\,dx$ **b)** $\displaystyle\int_1^4 5 + 2f(x)\,dx$ **c)** $\displaystyle\int_{-4}^4 f(x) + 3\,dx$ **d)** $\displaystyle\int_{-1}^2 f(x)\,dx$



$\int_0^1 f(x)\,dx = 0.4$
$\int_0^2 f(x)\,dx = 0.8$
$\int_0^3 f(x)\,dx = 0.9$
$\int_0^4 f(x)\,dx = 1.0$

4. Review all three previous Practests.

5. Find the arc length of $\ln(\cos x)$ on the interval $[0, \pi/3]$. Ans: $\ln|2 + \sqrt{3}|$

6. Determine whether the alternating series $\displaystyle\sum_{n=2}^{\infty}(-1)^n\left(\frac{\sqrt{n}}{n-1}\right)$ converges. Carefully check whether $a_{n+1} \le a_n$.

7. Integral Mix Up: First classify each by the technique that you think will apply: substitution, parts, parts twice, or ordinary methods. (Trig sub covered elsewhere.)

a) $\displaystyle\int 2e^{-3x}\,dx$ **b)** $\displaystyle\int \sec^2 x e^{\tan x}\,dx$ **c)** $\displaystyle\int e^x \cos x\,dx$

d) $\displaystyle\int x\sec^2 x\,dx$ **e)** $\displaystyle\int \cos(2\pi x)\,dx$ **f)** $\displaystyle\int \cos^2(\pi x)\,dx$

g) $\displaystyle\int (2x^2 + x)e^x\,dx$ **h)** $\displaystyle\int \arctan x\,dx$ **i)** $\displaystyle\int x^2 \ln x\,dx$

j) $\displaystyle\int x\sqrt{x-1}\,dx$ **k)** $\displaystyle\int \sec 3x\,dx$ **l)** $\displaystyle\int \sec^3 2x\,dx$

m) Censored **n)** $\displaystyle\int \frac{x}{25 + x^2}\,dx$ **o)** $\displaystyle\int \frac{1}{1 + 25x^2}\,dx$

p) $\displaystyle\int \cos^3 2x\,dx$ **q)** $\displaystyle\int \tan^4 \pi x\,dx$ **r)** $\displaystyle\int \sin^2 2\pi x\,dx$

s) $\displaystyle\int \frac{1}{\sqrt{1 - 25x^2}}\,dx$ **t)** $\displaystyle\int \frac{\cos x}{\sqrt{1 - \sin^2 x}}\,dx$ **u)** $\displaystyle\int \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\,dx$

8. **a)** Determine $\int (\ln x)^2\,dx$.

b) Let R be the region enclosed by $y = \ln x$, the x-axis, and $x = e$ in the first quadrant. Rotate R about the x-axis and find the volume.

9. **a)** Assume that n is a positive integer. Find $\int x^n \ln x\,dx$.

b) Find $\int \ln(\sqrt[n]{x})\,dx$.

10. Let R be the two-part region enclosed by $y = \cos \pi x$, the x-axis, $x = 0$, and $x = 1$. Rotate R about the x-axis. Find the volume of the resulting solid. (Ans: $\pi/2$)

11. Let R be the region enclosed by $y = \cos x$, the x-axis, $x = 0$, and $x = \pi/2$ in the first quadrant. Rotate R about the y-axis. Find the volume of the resulting solid using shells. (Ans: $\pi^2 - 2\pi$)

12. Evaluate these limits.

a) $\displaystyle\lim_{x\to 0} \frac{e^x - \cos x}{2x^3 + 2x}$ **b)** $\displaystyle\lim_{x\to\infty} \frac{x^2 + 7x}{9x^2 + 1}$ **c)** $\displaystyle\lim_{x\to 0} \frac{\cos x + \sin x}{3x + 2}$ **d)** $\displaystyle\lim_{x\to 0} \frac{x\cos x}{x^2 + 2x}$

e) $\displaystyle\lim_{x\to\infty} \frac{x\ln x}{e^x}$ **f)** $\displaystyle\lim_{x\to 0} \frac{\cos 2x - \cos 4x}{x^2}$ **g)** $\displaystyle\lim_{x\to 0} \frac{\arctan x}{\sin 5x}$ **h)** $\displaystyle\lim_{x\to 0^+} 2x\ln x$

i) $\displaystyle\lim_{x\to\infty} x^2 e^{-x}$ **j)** $\displaystyle\lim_{x\to\infty} \ln(2x+9) - \ln(x+7)$ **k)** $\displaystyle\lim_{x\to\infty} (1 + \tfrac{2}{x})^x$ **l)** $\displaystyle\lim_{n\to\infty} \sqrt[n]{n}$

m) $\displaystyle\lim_{n\to\infty} (1 + 3n)^{1/n^2}$

13. Try the following.

a) $\displaystyle\int_0^\infty \frac{4}{4+x^2}\,dx$ **b)** $\displaystyle\int_3^\infty \frac{4}{4-x^2}\,dx$ **c)** Censored **d)** $\displaystyle\int_3^\infty \frac{4x}{4-x^2}\,dx$

e) $\displaystyle\int \frac{4}{\sqrt{4+x^2}}\,dx$ **f)** $\displaystyle\int \frac{4}{\sqrt{4-x^2}}\,dx$ **g)** $\displaystyle\int \frac{4x}{(4+x^2)^{3/2}}\,dx$ **h)** $\displaystyle\int \frac{4x^2}{\sqrt{4-x^2}}\,dx$

i) $\displaystyle\int_0^2 \sqrt{4-x^2}\,dx$ **j)** $\displaystyle\int \frac{4x+1}{x^2-5x+4}\,dx$ **k)** $\displaystyle\int \frac{4x}{\sqrt{x-4}}\,dx$ **l)** $\displaystyle\int \frac{4x+8}{x^2+4x+5}\,dx$

m) $\displaystyle\int \frac{-4x+4}{(x-2)^2 x}\,dx$ **n)** $\displaystyle\int_2^4 \frac{\sqrt{x^2-4}}{x}\,dx$ **o)** Dropped **p)** $\displaystyle\int \frac{4}{(4-x)^{2/3}}\,dx$

q) $\displaystyle\int \frac{4}{(4-x^2)^{3/2}}\,dx$ **r)** $\displaystyle\int \sin^3 \pi x\,dx$ **s)** $\displaystyle\int \cos^3 x \sin^2 x\,dx$ **t)** $\displaystyle\int \frac{-5x-3}{x^2-3x}\,dx$

u) $\displaystyle\int \frac{8x+4}{x^3+x^2-2x}\,dx$ **v)** $\displaystyle\int \frac{4x^2+8x+2}{x(x+1)^2}$ **w)** $\displaystyle\int \sin^2 x + \cos^2 x\,dx$

14. Find the average value of $f(x) = \frac{2}{x^2+12x+35}$ on $[-1, 1]$. (Ans: $\frac{1}{2}(2\ln 6 - \ln 4 - \ln 8)$.)

15. Find the limit of each of these sequences, if it exists.

a) $\displaystyle\left\{\frac{3+2\sqrt{n}}{1+\sqrt{n}}\right\}_{n=1}^{\infty}$ **b)** $\displaystyle\left\{\frac{3+2n}{1+\sqrt{n}}\right\}_{n=1}^{\infty}$ **c)** $\{2\arctan(n+2)\}_{n=1}^{\infty}$ **d)** $\{\ln(2n) - \ln(n+1)\}_{n=1}^{\infty}$

16. Does the series $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$ converge? Explain.

17. **a)** Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$ converge?

b) Do it again using another test.

c) List two other tests that could also be used.

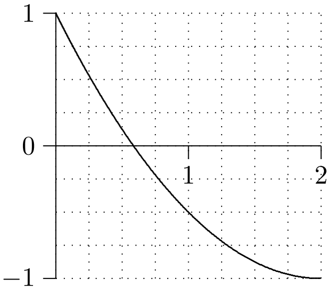
18. **a)** Carefully state the Mean Value Theorem and draw a figure that illustrates it.

b) Name two instances where we used the Mean Value Theorem this term!

19. Know the summation formula for $\sum_{k=1}^{n} k$ and $\sum_{k=1}^{n} k^2$

20. a) From a Final Exam: Draw and then estimate Right(4) for the graph of f on $[0,2]$ below. Be careful about how you draw your rectangles. Watch the scale. How many rectangles should you draw?

b) Is your estimate an over-estimate or an underestimate? Explain why.



21. Suppose that $f(x) = x^3 - 2x$ on $[0,2]$.

a) Compute Right(n) for this situation.

b) Use your Riemann sum to find $\int_0^2 x^3 - 2x\,dx$. Then check your answer by using antidifferentiation.

22. Assume f and g are continuous and that $\int_{-4}^{6} f(x)\,dx = 27$, $\int_0^2 f(x)\,dx = 12$, $\int_2^6 f(x)\,dx = 20$, and $\int_{-4}^{6} g(x)\,dx = -12$. Evaluate the following.

a) $\displaystyle\int_0^6 f(x)\,dx$ **b)** $\displaystyle\int_{-4}^0 f(x)\,dx$ **c)** $\displaystyle\int_0^{-4} f(x)\,dx$

d) $\displaystyle\int_{-1}^1 f(x+1)\,dx$ **e)** $\displaystyle\int_3^3 x^2 f(x)\,dx$ **f)** $\displaystyle\int_{-4}^6 (f(x) - 4g(x))\,dx$

23. a) If $f(x) = \int_1^x \sin(e^t)\,dt$, what is $f'(x)$? What fundamental theorem did you use?

b) If $f(x) = \int_x^2 e^{\sin t}\,dt$, what is $f'(x)$?

c) Suppose that $x^2 \sin(\pi x) = \int_0^x g(t)\,dt$. Evaluate $g(1)$ and explain your answer. Hint: How can you first find $g(x)$?

24. Evaluate $\displaystyle\int_e^{e^2} \frac{1}{t \ln t}\,dt$. (Ans: $\ln 2$.)

25. On the final exam for Math 131, Jody says that $\displaystyle\int \cos(\sqrt{x})\,dx = 2\cos(\sqrt{x}) + 2\sqrt{x}\sin(\sqrt{x}) + c$. Determine whether she should receive credit for her answer. Explain. Hint: I do not expect that you can do this integration.

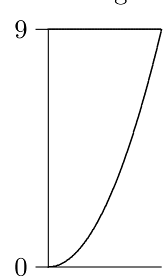
26. Find the area of the wedge-shaped region *below* the curves $y = \sqrt{x-1}$, $y = 3 - x$, and above the x-axis. Integrate along either axis: your choice! (Ans: 7/6.)

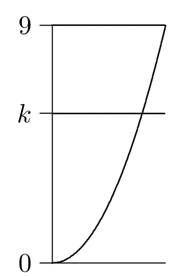
27. Find the area enclosed by the curves $y = x^2 + 2x$ and $y = x^3 - 4x$. Draw the figure. (Ans: $21\frac{1}{12}$)

28. Find the area of the region bounded by $y = \arcsin x$, the x axis and the line $x = 1$ in the first quadrant. Hint: You can switch axes or do integration by parts. (Ans: $\frac{1}{2}\pi - 1$.)

29. a) The region R in the first quadrant enclosed by $y = x^2$, the y-axis, and $y = 9$ is shown in the graph on the left below. Find the area of R.

b) A horizontal line $y = k$ is drawn so that the region R is divided into two pieces of equal area. Find the value of k. (See the graph on the right). Hint: It might be easier to integrate along the y-axis now.





30. a) Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$ and $y = x$ is revolved about the x-axis. (Answer: $\pi/6$)

b) Find the volume of the solid that results when the region in the first quadrant enclosed by $y = \sqrt{9 - x^2}$, $y = 1$, $y = 3$, and the y axis is revolved about the y-axis. (Answer: $28\pi/3$)

31. A small canal buoy is formed by taking the region in the first quadrant bounded by the y-axis, the parabola $y = 2x^2$, and the line $y = 5 - 3x$ and rotating it about the y-axis. (Units are feet.) Find the volume of this buoy. (Answer: 2π cubic feet.)

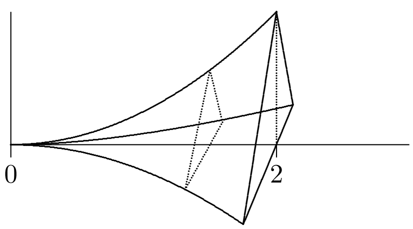
32. See Lab 6 if you need practice on Volume Problems.

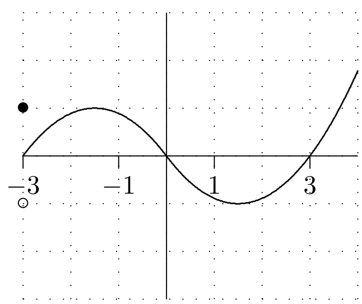
33. Find the average value of $f(x) = \dfrac{1}{\sqrt{4 - 36x^2}}$ on the interval $[0, \frac{1}{6}]$. (Ans: $\pi/6$)

34. a) A tank is formed by rotating the region between $y = x^2$, the y-axis and the line $y = 4$ in the first quadrant around the y axis. The oil in the tank has density 50 lbs/ft^3. Find the work done pumping the oil to the top of the tank if there is only 1 foot of oil in the tank.

b) Suppose the tank is empty and is **filled** from a hole in the bottom to a depth of 3 feet. Find the work done.

35. A crystal prism is 2 cm long (below left). Its cross-sections are isosceles triangles whose bases are twice the heights. If the heights are formed by the curve $y = \frac{x^2}{4}$, find the volume of the prism. (Answer: 0.4 cu. cm.)





36. Let $F(x)$ be the antiderivative of $f(x)$ on $[-3, 4]$, where f is the function graphed above (right). Since F is an antiderivative of f, then $F' = f$. Use this relationship to answer the following questions.

a) On what interval(s) is F increasing? Decreasing?

b) At what point(s), if any, does F have a local max? Min?

c) On what interval(s) is F concave up? Down?

d) Does F have any points of inflection?

e) Assume F passes through the point $(-3, 1)$ indicated with a $\bullet$; draw a potential graph of F.

f) Assume, instead, that F passes through $(-3, -1)$ indicated by a $\circ$; draw a graph of F. What is the relationship between the two graphs you've drawn?

37. Find the interval of convergence for these power series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-3)^n}{5^{n+1}}$ **b)** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3x^n}{n!}$ **c)** $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3n-2}$ **d)** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-5)^n}{n^2+1}$

e) $\displaystyle\sum_{n=0}^{\infty} (-1)^n n^2 x^n$ **f)** $\displaystyle\sum_{n=0}^{\infty} 9^n x^{2n}$ **g)** $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)!(x+4)^n}{(2n)!}$ **h)** $\displaystyle\sum_{n=0}^{\infty} \frac{n^n x^n}{n!}$ (R only)

38. Determine whether the following series converge. Justify your answers with an argument.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[7]{n^5+4n}}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$ **c)** $\displaystyle\sum_{n=1}^{\infty} \frac{301n}{(n+101)2^n}$ **d)** $\displaystyle\sum_{n=1}^{\infty} \frac{n^6+4}{n^7}$ **e)** $\displaystyle\sum_{n=1}^{\infty} e\left(\frac{\pi}{2!}\right)^{2!n}$

f) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-5n+4}$ **g)** $\displaystyle\sum_{n=1}^{\infty} \sin\frac{1}{n^3}$ **h)** $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n^2+1}$ **i)** $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2-1}{2n^9+1}$ **j)** $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{7^n+2}$

39. Determine whether these alternating series converge conditionally, absolutely, or not at all. **Justify your answers your answers with a complete argument.**

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[7]{n^5+4n}}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(21n+1)}{31n+2}$ **c)** $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{2^n}$ **d)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n^3+1}}$

Math 131 Final Review Answers

1. See the answers to Lab 14 on line.

2. Use antiderivatives: $\int_1^8 F'(x)\,dx = F(x)\Big|_1^8 = F(8) - F(1) = 4 - 1 = 3$

3. The answers are:

a) $-\int_0^3 f(x)\,dx = -0.9$ **b)** $\int_1^4 5\,dx + \int_1^4 f(x)\,dx = 15 + 2(0.6)$ **c)** $\int_{-4}^4 f(x)\,dx + \int_{-4}^4 3\,dx = 0 + 24$

d) $= \int_{-1}^0 f(x)\,dx + \int_0^2 f(x)\,dx$ symmetry $= -\int_0^1 f(x)\,dx + \int_0^2 f(x)\,dx = -0.4 + 0.8 = .4$

5. $AL = \int_0^{\pi/3} \sqrt{1+\tan^2 x}\,dx = \int_0^{\pi/3} \sec x\,dx = \ln|\sec x + \tan x|\Big|_0^{\pi/3}$.

6. Use the alternating series test. Check the two conditions.

1) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n}}{n-1} = \lim_{n\to\infty} \sqrt{\frac{n}{(n-1)^2}} = \lim_{n\to\infty} \sqrt{\frac{n}{n^2-2n+1}} = \lim_{n\to\infty} \sqrt{\frac{1/n}{1-2/n+1/n^2}} = 0$ ✓.

2) Decreasing? Let $f(x) = \frac{\sqrt{x}}{x-1}$. Then

$$f'(x) = \frac{\frac{x-1}{2\sqrt{x}} - \sqrt{x}}{(x+1)^2} = \frac{\frac{x-1-2x}{2\sqrt{x}}}{(x+1)^2} = \frac{\frac{-x-1}{2\sqrt{x}}}{(x+1)^2} < 0, \qquad (x \geq 2)$$

so the function and the sequence are decreasing.✓So by the AST, the series converges.

7. Brief Answers to the Mix-Up Problem: (All "$+c$".)

a) $-\frac{2}{3}e^{-3x}$ **b)** $e^{\tan x}$ **c)** $\frac{1}{2}e^x(\cos x + \sin x)$
d) $x\tan x - \ln|\sec x|$ **e)** $\frac{1}{2\pi}\sin(2\pi x)$ **f)** $\frac{1}{2}x + \frac{1}{4\pi}\sin 2\pi x$
g) $(2x^2 - 3x + 3)e^x$ **h)** $x\arctan x - \frac{1}{2}\ln|x^2+1|$ **i)** $\frac{1}{3}x^3\left(\ln x - \frac{1}{3}\right)$
j) $\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2}$ **k)** $\frac{1}{3}\ln|\sec 3x + \tan 3x|$ **l)** $\frac{1}{4}(\sec 2x\tan 2x + \ln|\sec 2x + \tan 2x|)$
m) Censored **n)** $\frac{1}{2}\ln(25+x^2)$ **o)** $\frac{1}{5}\arctan 5x$
p) $\frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x$ **q)** $\frac{1}{3\pi}\tan^3 \pi x - \frac{1}{\pi}\tan \pi x + x$ **r)** $\frac{x}{2} - \frac{1}{8\pi}\sin 4\pi x + c$
s) $\frac{1}{5}\arcsin 5x$ **t)** $\arcsin(\sin x)$ **u)** $\frac{1}{2}(\arcsin x)^2$

8. a) Use parts twice, the first time $u = (\ln x)^2$ and the second time $u = 2\ln x$. Ans: $x(\ln x)^2 - 2x\ln x + 2x + c$.

b) From (a): $\int_1^e \pi(\ln x)^2\,dx = \pi\left[x(\ln x)^2 - 2x\ln x + 2x\right]\Big|_1^e = \pi(e-2)$.

9. a) $\frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + c$

b) $\frac{1}{n}(x\ln x - x)$. Hint: $\ln(\sqrt[n]{x}) = \ln(x^{1/n}) = \frac{1}{n}\ln x$.

10. The two pieces are symmetric (so double one of them). Use half-angle formula: $V = 2\cdot\int_0^{1/2}\pi\cos^2\pi x\,dx = 2\pi\int_0^{1/2}\frac{1}{2} + \frac{1}{2}\cos(2\pi x)\,dx = 2\pi\left(\frac{1}{2}x + \frac{1}{4\pi}\sin(2\pi x)\right)\Big|_0^{1/2} = \frac{\pi}{2}$.

11. Use parts with $u = x$. $V = \int_0^{\pi/2} 2\pi x\cos x\,dx = 2\pi[x\sin x - \int_0^{\pi/2}\sin x\,dx] = 2\pi[x\sin x + \cos x]\Big|_0^{\pi/2}$.

12. L'H once for a, d, g. Twice for b, e, f. Convert h and i to quotients. Use log laws for j. Key limits for k and l. For m: take log and convert to quotient.

a) 1/2 **b)** 1/9 **c)** 1/2 **d)** 1/2 **e)** 0 **f)** 6
g) 1/5 **h)** 0 **i)** 0 **j)** $\ln 2$ **k)** e^2 **l)** 1 **m)** 1

13. a) $\lim_{b\to\infty}\int_0^b \frac{1}{1+\frac{x^2}{4}}\,dx = \lim_{b\to\infty} 2\arctan\frac{x}{2}\Big|_0^b = \lim_{b\to\infty} 2\arctan\frac{b}{2} - 0 = 2(\frac{\pi}{2}) - 0 = \pi$.

b) $\lim_{b\to\infty}\int_3^b \frac{4}{4-x^2}\,dx = \lim_{b\to\infty}\int_3^b \frac{1}{2-x} + \frac{1}{2+x}\,dx = \lim_{b\to\infty}(-\ln|2-x| + \ln|2+x|)\Big|_3^b = \lim_{b\to\infty}\ln|\frac{2+x}{2-x}|\Big|_3^b = \lim_{b\to\infty}\ln|\frac{2+b}{2-b}| - \ln 5 = \ln 1 - \ln 5 = -\ln 5$.

d) $\lim_{b\to\infty}\int_3^b \frac{4x}{4-x^2}\,dx = \lim_{b\to\infty}(-2\ln|4-x^2|)\Big|_3^b = \lim_{b\to\infty}\ln|4-b^2| - \ln 5 = +\infty$. Diverges.

e) With triangles where $x = 2\tan\theta$: $\int \frac{4}{\sqrt{4+x^2}}\,dx = \int \frac{4}{2\sec^2\theta} 2\sec^2\theta\,d\theta = \int 4\sec\theta\,d\theta = 4\ln|\sec\theta+\tan\theta|+c = 4\ln\left|\frac{\sqrt{4+x^2}+x}{2}\right|+c$

f) Similar to the one above. $\int \frac{4}{\sqrt{4-x^2}}\,dx = \int \frac{4}{2\sqrt{1-\frac{1}{4}x^2}}\,dx = 4\arcsin(\frac{x}{2})+c$.

g) With triangles: $\int \frac{8\tan\theta}{8\sec^3\theta} 2\sec^2\theta\,d\theta = \int 2\sin\theta\,d\theta = -2\cos\theta + c = -\frac{4}{\sqrt{4+x^2}}$

g) Much easier by u-sub: $= \int 2u^{-3/2}\,du = \int -4u^{-1/2} + c = -4(4+x^2)^{-1/2}+c$.

h) $\int \frac{16\sin^2\theta}{2\cos\theta}2\cos\theta\,d\theta = \int 16\sin^2\theta\,d\theta = -8\sin\theta\cos\theta+8\theta+c = -2x\sqrt{4+x^2}+8\arcsin(\frac{x}{2})+c$

i) $\int_0^{\pi/2} 2\cos\theta 2\cos\theta\,d\theta = \int_0^{\pi/2}4\cos^2\theta\,d\theta = 2\cos\theta\sin\theta+2\theta\Big|_0^{\pi/2} = \pi$.

j) $\int \frac{17/3}{x-4} - \frac{5/3}{x-1}\,dx = \frac{17}{3}\ln|x-4| - \frac{5}{3}\ln|x-1|+c$.

k) $u = x-4$. So $\int \frac{4u+16}{u^{1/2}}\,du = \int 4u^{1/2}+16u^{-1/2}\,du = \frac{8}{3} u^{3/2}+32u^{1/2}+c = \frac{8}{3}(x-4)^{3/2}+32\sqrt{x-4}+c$.

l) $\int\frac{4x+8}{x^2+4x+5}\,dx = 2\ln|x^2+4x+5|+c$. (Use u-substitution.)

m) $\int -\frac{1}{x-2} - \frac{2}{(x-2)^2}+\frac{1}{x}\,dx = -\ln|x-2| + 2(x-2)^{-1}+\ln|x|+c$.

n) $\int_0^{\pi/3}\frac{2\tan\theta}{2\sec\theta}2\sec\theta\tan\theta\,d\theta = 2\int_0^{\pi/3}\tan^2\theta\,d\theta = 2\int_0^{\pi/3}\sec^2\theta - 1\,d\theta = 2\tan\theta - 2\theta\Big|_0^{\pi/3} = 2\sqrt{3} - \frac{2}{3}\pi$.

o) Dropped

p) $\int -4u^{-2/3}\,du = -12u^{1/3}+c = -12(4-x)^{-1/3}+c$.

q) $4\int\frac{2\cos\theta}{8\cos^3\theta}\,d\theta = \int\sec^2\theta\,d\theta = \tan\theta + c = \frac{x}{\sqrt{4-x^2}}+c$.

r) Use trig id and then u sub with $u=\cos\pi x$: $=\int\sin\pi x(1-\cos^2\pi x)\,dx = \int \sin\pi x - \cos^2\pi x\sin\pi x\,dx = -\frac{1}{\pi}\cos\pi x + \frac{1}{3\pi}\cos^3(\pi x)+c$

s) $= \int\cos x(\cos^2 x)\sin^2 x\,dx = \int\cos x(1-\sin^2 x)\sin^2 x\,dx = \int\cos x(\sin^2 x - \sin^4 x)\,dx$. Use $u = \sin x$. We get $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c$.

t) Partial Fractions: $\int\frac{8x+4}{x^3+x^2-2x}\,dx = \int\frac{4}{x+1}-\frac{2}{x+2}-\frac{2}{x}\,dx = 4\ln|x+1|-2\ln|x+2|-2\ln|x|+c$.

u) Partial Fractions: $\int\frac{-5x-3}{x^2-3x}\,dx = \int\frac{1}{x} - \frac{6}{x-3}\,dx = \ln|x| - 6\ln|x-3|+c$.

v) $\int\frac{4x^2+8x+2}{x(x+1)^2}\,dx = \int\frac{2}{x}+\frac{2}{x+1}+\frac{2}{(x+1)^2}\,dx = 2\ln|x|+2\ln|x+1|-2(x+1)^{-1}+c$.

w) $\int\sin^2 x+\cos^2 x\,dx = \int 1\,dx = x+c$.

14. Use partial fractions $\frac{1}{1-(-1)}\int_{-1}^{1}\frac{1}{x+5}-\frac{1}{x+7}\,dx = \frac{1}{2}(\ln|x+5|-\ln|x+7|)\Big|_{-1}^{1}$.

15. Use L'H for a and b. For d use $\ln\frac{2n}{n+1} = \ln\frac{2}{1+\frac{1}{n}}$.

a) 2 **b)** ∞ **c)** π **d)** $\ln 2$

16. Using the ratio test test the series converges because $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{n+1}{e^{n+1}}\cdot\frac{n}{e^n} = \lim_{n\to\infty}\frac{n+1}{n}\cdot\frac{1}{e} = \frac{1}{e} < 1$

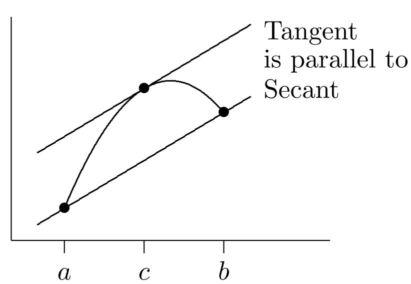
17. a) Since $\frac{1}{n^2+5n+6} < \frac{1}{n^2}$ and $\sum\frac{1}{n^2}$ converges by p-series test ($p=2>1$), so $\sum\frac{1}{n^2+5n+6}$ by Direct Comparison.

b) Use the telescoping sum test (partial fractions): $\sum_{n=1}^{\infty}\frac{1}{n^2+5n+6} = \sum_{n=1}^{\infty}\frac{1}{n+2} - \frac{1}{n+3}$. Show that $s_n = \frac{1}{3} - \frac{1}{n+3}$. So $\lim_{n\to\infty}s_n = \frac{1}{3}$ and so the series converges to $\frac{1}{3}$.

c) Use the limit comparison test with $\sum\frac{1}{n^2}$ or use the integral test with partial fractions: $\int_1^{\infty}\frac{1}{x^2+5x+6}\,dx = \lim_{b\to\infty}\ln\left|\frac{x+2}{x+3}\right|\Big|_1^b = -\ln|\frac{3}{4}|$. Converges.

18. a) Let f be a continuous function on the closed interval $[a,b]$ and differentiable on the open interval $[a,b]$. Then there is some point c between a and b so that

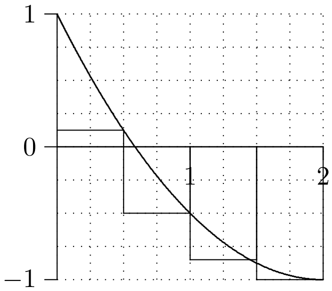
$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$



b) In the proof of the Fundamental Theorem of Calculus and the proof of the arc length formula, for example.

19. Look them up.

20. Right(4) ≈ -1.1 and it is an underestimate because f is decreasing.



21. $\Delta x = \frac{2-0}{n} = \frac{2}{n}$, $x_k = \frac{2k}{n}$, $f(x_k) = (\frac{2k}{n})^3 - \frac{4k}{n}$. Right$(n)\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}\left[\left(\frac{2k}{n}\right)^3 - \frac{4k}{n}\right]\frac{2}{n}$. Now use summation formulas to simplify Right(n).

$$\text{Right}(n) = \frac{2}{n}\sum_{k=1}^{n}\left(\frac{2k}{n}\right)^3 - \frac{2}{n}\sum_{k=1}^{n}\frac{4k}{n} = \frac{16}{n^4}\sum_{k=1}^{n}k^3 - \frac{8}{n^2}\sum_{k=1}^{n}k = \frac{16}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{8}{n^2}\left[\frac{n(n+1)}{2}\right] = \frac{4(n+1)^2}{n^2} - \frac{4n+4}{n}.$$

Finally, to get the exact "area" evaluate

$$\lim_{n\to\infty}\text{Right}(n) = \lim_{n\to\infty}\frac{4(n+1)^2}{n^2} - \frac{4n+4}{n} = \lim_{n\to\infty}\frac{4n^2+8n+4}{n^2} - \frac{4+\frac{4}{n}}{1} = \lim_{n\to\infty}\frac{4+\frac{8}{n}+\frac{4}{n^2}}{1} - 4 - \frac{4}{n} = 0.$$

22. **a)** $\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx = 12 + 20 = 32.$
b) $\int_{-4}^0 f(x)\,dx = \int_{-4}^6 f(x)\,dx - \int_0^6 f(x)\,dx = 27 - 32 = -5.$
c) $\int_0^{-4} f(x)\,dx = -\int_{-4}^0 f(x)\,dx = -(-5) = 5.$
d) $y = f(x+1)$ is a horizontal shift of $y = f(x)$ one unit to the left, so $\int_{-1}^1 f(x+1)\,dx = \int_0^2 f(x)\,dx = 12.$
e) $\int_3^3 x^2 f(x)\,dx = 0$ because the endpoints are the same.
f) $\int_{-4}^6 (f(x) - 4g(x))\,dx = \int_{-4}^6 f(x)\,dx - 4\int_{-4}^6 g(x)\,dx = 27 - 4(-12) = 75.$

23. **a)** $f'(x) = \sin(e^x)$ by the Fundamental Theorem of Calculus.
b) Switch order of limits: then $f'(x) = -e^{\sin x}$
c) Take the derivative of each side: Then $\pi x^2\cos(\pi x) + 2x\sin(\pi x) = g(x)$. So $g(1) = -\pi$.

24. Use substitution with $u = \ln t$ and $du = \frac{1}{t}dt$. $x = e \Rightarrow u = 1$ and $x = e^2 \Rightarrow u = 2$. So $\int_1^2 \frac{1}{u}\,du = \ln|u|\Big|_1^2$.

25. Jody gets credit. Check that her answer is correct by taking the dervative.

26. Use $\int_0^1 \sqrt{x-1}\,dx + \int_1^3 3 - x\,dx = \frac{2}{3}(x-1)^{3/2}\Big|_1^2 + 3x - \frac{1}{2}x^2\Big|_2^3$.

27. Use $\int_{-1}^0 (x^3 - 4x) - (x^2 + 2x)\,dx + \int_0^3 (x^2 + 2x) - (x^3 - 4x)\,dx$

29. **a)** $A = \int_0^3 9 - x^2\,dx = 9x - \frac{1}{3}x^3\Big|_0^3 = (27 - 9) - 0 = 18.$
b) Note: The intersection point of the horizontal line $y = k$ with the parabola is at $(\sqrt{k}, k)$. Find k so that $\frac{A}{2} = \frac{18}{2} = 9 = \int_0^{\sqrt{k}} k - x^2\,dx = kx - \frac{1}{3}x^3\Big|_0^{\sqrt{k}} = k^{3/2} - \frac{1}{3}k^{3/2} = \frac{2}{3}k^{3/2}$. Therefore, $k^{3/2} = \frac{3}{2}\cdot 9 = 13.5 \Rightarrow k = 13.5^{2/3} \approx 5.6696$.

30. **a)** Use $\int_0^1 \pi(x - x^2)\,dx$.
b) $x = \sqrt{9 - y^2}$. Use disks around y-axis. $\int_1^3 \pi(9 - y^2)\,dy$. Or shells $\int_0^{\sqrt{8}} 2\pi x(\sqrt{9 - x^2} - 1)\,dx$.

31. $V = \int_0^1 2\pi x[(5 - 3x) - 2x^2]\,dx$.

33. $AV = \frac{1}{1/6 - 0}\int_0^{1/6}\frac{1}{\sqrt{4 - 36x^2}}\,dx = \frac{1}{1/6}\int_0^{1/6}\frac{1}{\sqrt{a^2 - u^2}}\,dx = \frac{1}{6}\cdot\frac{1}{6}\arcsin\frac{6x}{2}\Big|_0^{1/6} = \frac{1}{36}[\arcsin(1/2) - \arcsin(0)] = \pi/36$. Note: $u = 6x$ here so $du = 6dx$ or $\frac{1}{6}du = dx$.

34. Use $V = \int_0^2 \frac{1}{2}\frac{x^2}{4}\frac{2x^2}{4}\,dx = \int_0^2 \frac{x^4}{16}\,dx$

35. **a)** Work $= D\displaystyle\int_a^b A(y)[H-y]\,dy = 50\int_0^1 \pi(y)[4-y]\,dy = 50\pi\left(2y^2 - \frac{y^3}{3}\right)\Bigg|_0^1 = 50\pi\left[\left(2-\frac{1}{3}\right)\right] = \frac{250\pi}{3}$ ft-lbs.

b) Work $= D\displaystyle\int_a^b A(y)[H-y]\,dy = 50\int_0^3 \pi(y)[y-0]\,dy = 50\pi\left(\frac{y^3}{3}\right)\Bigg|_0^3 = 50\pi\left[\left(2-\frac{1}{3}\right)\right] = 450\pi$ ft-lbs.

36. Use that $F' = f$ and $F'' = f'$.

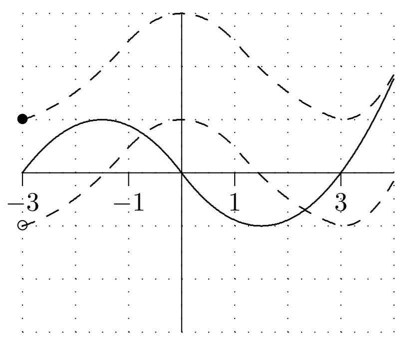
a) F increasing means $F' = f > 0$: on $(-3,0)$ and $(3,4)$. F decreasing means $F' = f < 0$: on $(0,3)$.

b) F has a local max means $F' = f$ changes from $+$ to $-$: at $x = 0$. F has a local min means $F' = f$ changes from $-$ to $+$: at $x = 3$.

c) F concave up means $F'' = f' > 0$, i.e., f is increasing: $(-3,-1.5)$ and $(1.5,4)$. F concave down means $F'' = f' < 0$, i.e., f is decreasing: $(-1.5,1.5)$.

d) F has a points of inflection when $F'' = f'$ changes sign, i.e., when f changes from increasing to decreasing or *vice versa*: at $x = \pm 1.5$.

f) The two graphs are parallel (differ only by a constant).



37. In each of these use the ratio test to find the radius of convergence R and then check the two endpoints. These answers are very brief. Yours should be more complete.

a) The center $a = 3$. After simplifying $r = \lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\frac{(n+1)(x-3)}{5n}\right| = \left|\frac{x-3}{5}\right| < 1 \iff |x-3| < 5$. $R = 5$. Check the endpoints. At $c + R = 3 + 5 = 8$: Simplifies to $\sum_{n=0}^{\infty}\frac{n5^n}{5^{n+1}} = \sum_{n=0}^{\infty}\frac{n}{5}$. Notice $\lim\limits_{n\to\infty}\frac{n}{5} = \infty \neq 0$. By the n-th term test for divergence, the series diverges. At $c - R = 3 - 5 = -2$: $\sum_{n=0}^{\infty}\frac{n(-5)^n}{5^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^n n}{5}$ diverges by the n-th term test as above. Interval: $(-5,5)$.

b) The center $a = 0$. After simplifying $r = \lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\frac{x}{n+1}\right| = 0 < 1$; it converges for all x. $R = \infty$. Interval: $(-\infty,\infty)$.

c) The center $a = 0$. After simplifying $r = \lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\frac{(3n-2)x}{3n+1}\right| \overset{\text{HPwrs}}{=} |x| < 1$. $R = 1$. Check the endpoints. At $c + R = 1$: Using the limit comparison with $\sum\frac{1}{n}$ which diverges by p-series test ($p = 1 \leq 1$): $\lim\limits_{n\to\infty}\frac{a_n}{b_n} = \lim\limits_{n\to\infty}\frac{1}{3n-2}\cdot\frac{n}{1} = \frac{1}{3}$ so by LCT $\sum_{n=0}^{\infty}\frac{1}{3n-2}$ diverges. At $c - R = -1$: $\sum_{n=0}^{\infty}\frac{(-1)^n}{3n-2}$, so use the alternating series test. Two conditions: (1) $\lim\limits_{n\to\infty}\frac{1}{3n-2} = 0$ and (2) let $f(x) = (3x-2)^{-1}$ so $f'(x) = (-1)(3x-2)^{-2} < 0$ so the terms are decreasing. By the AST, the series converges at $x = -1$. Interval: $[-1,1)$.

d) The center $a = 5$. After simplifying $r = \lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\frac{(n^2+1)(x-5)}{(n+1)^2+1}\right| = \lim\limits_{n\to\infty}\left|\frac{(n^2+1)(x-5)}{n^2+2n+2}\right| \overset{\text{HPwrs}}{=} |x-5| < 1$. $R = 1$. Check the endpoints. At $c+R = 6$: $\sum_{n=0}^{\infty}\frac{(-1)^n}{n^2+1}$, so use the alternating series test. Two conditions: (1) Since $\lim\limits_{n\to\infty}\frac{1}{n^2+1} = 0$ and (2) we have $f(x) = (x^2+1)^{-1}$ so $f'(x) = -1(2x)(x^2+1)^{-2} < 0$ so the terms are decreasing. The series converges at $x = 6$ by the AST. At $c - R = 4$: $\sum_{n=0}^{\infty}\frac{1}{n^2+1}$ converges by direct comparison with $\sum\frac{1}{n^2}$: $0 < \frac{1}{n^2+1} < \frac{1}{n^2}$. Since $\sum\frac{1}{n^2}$ converges (p-series, $p = 2 > 1$) so does $\sum_{n=0}^{\infty}\frac{1}{n^2+1}$. Interval: $[-1,1]$.

e) The center $a = 5$. After simplifying $r = \lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\frac{(n+1)^2 x}{n^2}\right| = \lim\limits_{n\to\infty}\left|\frac{(n^2+2n+1)x}{n^2}\right| \overset{\text{HPwrs}}{=} |x| < 1$. $R = 1$. Check the endpoints. At $x = 1$: $\sum_{n=0}^{\infty}(-1)^n n^2$ diverges by the n-th term test: $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}(-1)^n n^2 \neq 0$. At $x = -1$: $\sum_{n=0}^{\infty} n^2$ diverges by the n-th term test again. Interval: $(-1,1)$.

f) The center $a = 0$. After simplifying $r = \lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}|9x^2| < 1 \iff |x^2| < \frac{1}{9} \iff |x| < \frac{1}{3}$. $R = \frac{1}{3}$. Check the endpoints. At $x = a + R = \frac{1}{3}$: $\sum_{n=0}^{\infty} 9^n\left(\frac{1}{3}\right)^{2n} = \sum_{n=0}^{\infty} 1$ diverges by the n-th term test since $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} 1 = 1 \neq 0$. At $x = a - R = -\frac{1}{3}$: $\sum_{n=0}^{\infty} 9^n\left(-\frac{1}{3}\right)^{2n} = \sum_{n=0}^{\infty} 1$ diverges as above. Interval: $\left(-\frac{1}{3},\frac{1}{3}\right)$.

g) The center $a = -4$. After simplifying $r = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+2)(x+4)}{(2n+1)(2n+2)}\right| = \lim_{n\to\infty} \left|\frac{(n+2)x}{4n^2+6n+2}\right| \overset{\text{HPwrs}}{=} \lim_{n\to\infty} \left|\frac{nx}{4n^2}\right| = \lim_{n\to\infty} \left|\frac{x}{4n}\right| = 0$. $R = \infty$. Interval: $(-\infty, \infty)$.

h) $\sum_{n=1}^{\infty} \frac{n^n x^n}{n!}$ After simplifying

$$r = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)^{n+1}x^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n x^n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)^{n+1}x}{(n+1)n^n}\right|$$
$$= \lim_{n\to\infty} \left|\frac{(n+1)^n x}{n^n}\right|$$
$$= \lim_{n\to\infty} \left|\left(\frac{n+1}{n}\right)^n \cdot x\right| = \lim_{n\to\infty} \left|\left(1+\frac{1}{n}\right)^n \cdot x\right| = |ex|.$$

By the ratio test extension, the series *converges* if $|ex| < 1 \iff |x| < \frac{1}{e}$. The radius of convergence is $R = \frac{1}{e}$.

38. a) ARGUMENT: $\frac{1}{\sqrt[7]{n^5+4n}}$ and $\frac{1}{n^{5/7}}$ are both positive. Apply the LCT.

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\sqrt[7]{n^5+4n}} \cdot \frac{n^{5/7}}{1} \overset{\text{HPwrs}}{=} \lim_{n\to\infty} \frac{n^{5/7}}{\sqrt[7]{n^5}} = 1.$$

Since $0 < L = 1 < \infty$, and since $\sum_{n=1}^{\infty} \frac{1}{n^{5/7}}$ diverges by the p-series test ($p = \frac{2}{3} \le 1$), then $\sum_{n=1}^{\infty} \frac{1}{\sqrt[7]{n^5+4n}}$ diverges by the limit comparison test.

b) ARGUMENT: Limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (Avoid integral test.) For all n $\frac{\arctan n}{1+n^2}$ and $\frac{1}{n^2}$ are positive. Apply the LCT.

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\arctan n}{1+n^2} \cdot \frac{n^2}{1} = \lim_{n\to\infty} \arctan n \cdot \frac{n^2}{n^2+1} = \lim_{n\to\infty} \arctan n \cdot \frac{1}{1+\frac{1}{n^2}} = \frac{\pi}{2}.$$

Since $0 < L = \frac{\pi}{2} < \infty$, and since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-series test ($p = 2 > 1$), then $\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$ converges by the limit comparison test.

c) ARGUMENT: LCT. $\frac{301n}{(n+101)2^n}$ and $\frac{1}{2^n}$ are both positive. Apply the Limit Comparison test.

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{301n}{(n+101)2^n} \cdot \frac{2^n}{1} = \lim_{n\to\infty} \frac{301n}{n+101} \overset{\text{HPwrs}}{=} \lim_{n\to\infty} \frac{301n}{n} = 301.$$

Since $0 < L = 301 < \infty$, and since $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges (geometric series test, $|r| = \frac{1}{2} < 1$), then $\sum_{n=1}^{\infty} \frac{301n}{(n+101)2^n}$ converges by the limit comparison test.

d) ARGUMENT: DCT. $\frac{n^6+4}{n^7} = \frac{1}{n} + \frac{4}{n^7} > \frac{1}{n} > 0$ and $\sum \frac{1}{n}$ diverges by the p-series test ($p = 1 \le 1$). So $\sum \frac{n^6+1}{n^7}$ diverges by direct comparison. (LCT works, too.)

e) ARGUMENT: Since $\frac{\pi}{2!} = \frac{\pi}{2} > 1$, then $|r| = (\frac{\pi}{2!})^{2!} > 1$. So rewriting the original series as the geometric series $\sum_{n=1}^{\infty} e\left[\left(\frac{\pi}{2!}\right)^{2!}\right]^n$, it diverges.

f) ARGUMENT: Limit comparison test with $\sum \frac{1}{n^2}$. For $n \ge 2$ we have $\frac{3}{n^2-3n+4}$ and $\frac{1}{n^2}$ are positive.

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2}{n^2-3n+4} \cdot \frac{n^2}{1} = \lim_{n\to\infty} \frac{2n^2}{n^2-3n+4} \overset{\text{HPwrs}}{=} \lim_{n\to\infty} \frac{2n^2}{n^2} = 2.$$

Since $0 < L = 2 < \infty$, and since $\sum \frac{1}{n^2}$ converges by the p-series test ($p = 2 > 1$), then $\sum \frac{2}{n^2-3n+4}$ converges by the limit comparison test.

g) ARGUMENT: Limit comparison test with $\sum \frac{1}{n^3}$. Since $0 < \frac{1}{n^3} < 1 < \pi/2$, then $\sin \frac{1}{n^3} > 0$. So both series have positive terms.

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin \frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{x\to\infty} \frac{\sin \frac{1}{x^3}}{\frac{1}{x^3}} \overset{\text{l'Ho}}{=} \lim_{x\to\infty} \frac{\cos(\frac{1}{x^3})(-\frac{3}{x^4})}{-\frac{3}{x^4}} = \lim_{x\to\infty} \cos \frac{1}{x^3} = \cos 0 = 1.$$

Since $0 < L = 1 < \infty$, and since $\sum \frac{1}{n^3}$ converges by the p-series test ($p = 3 > 1$), then $\sum \sin \frac{1}{n^3}$ diverges by the limit comparison test.

h) ARGUMENT: Diverges by the nth term test.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n}{n^2+1} = \lim_{x\to\infty} \frac{3^x}{x^2+1} \overset{\text{l'Ho}}{=} \lim_{x\to\infty} \frac{(\ln 3)3^x}{2x} \overset{\text{l'Ho}}{=} \lim_{x\to\infty} \frac{(\ln 3)^2 3^x}{2} = \infty \ne 0.$$

i) ARGUMENT: Limit comparison test with $\sum \frac{1}{n^7}$. Both series have positive terms.

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{5^2-1}{2n^9+1} \cdot \frac{n^7}{1} = \lim_{n\to\infty} \frac{5n^9-n^7}{2n^9+1} \overset{\text{HPwrs}}{=} \lim_{n\to\infty} \frac{5n^9}{2n^9} = \frac{5}{2} > 0.$$

Since $0 < L = 1 < \infty$, and since $\sum \frac{1}{n^7}$ converges by the p-series test ($p = 7 > 1$), then $\sum \frac{5n^2-1}{2n^9+1}$ converges by the limit comparison test.

j) ARGUMENT: Direct comparison test with $\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$. $0 \le \frac{2^n}{7^n+2} < \frac{2^n}{7^n} = \left(\frac{2}{7}\right)^n$. But $\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$ converges (geometric series, ($|r| = \frac{2}{7} < 1$), so then $\frac{2^n}{7^n+2}$ converges by the direct comparison test.

39. a) ARGUMENT: Check absolute convergence: $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{\sqrt[7]{n^5+4n}}\right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt[7]{n^5+4n}}$ which diverges by part (a) of the previous problem. Does not converge absolutely. Conditional convergence: Use the alternating series test with $a_n = \frac{1}{\sqrt[7]{n^5+4n}} > 0$. Check the two conditions.

1. $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{1}{\sqrt[7]{n^5+4n}} = 0.$✓

2. Decreasing? $a_{n+1} \le a_n \iff \frac{1}{\sqrt[7]{(n+1)^5+4(n+1)}} \le \frac{1}{\sqrt[7]{n^5+4n}}$ which is true since the denominator of a_{n+1} is greater than for a_n and the numerators are the same.✓By the Alternating Series test, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[7]{n^5+4n}}$ converges. Overall: Converges conditionally.

b) ARGUMENT: Use the alternating series test with $a_n = \frac{21n+1}{31n+2}$. Check the two conditions.

1. $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{21n+1}{31n+2} \overset{\text{HPwrs}}{=} \lim\limits_{n\to\infty} \frac{21n}{31n} = \frac{21}{31} \neq 0$. Fails. Since the first hypothesis is not satisfied, the alternating series test does not apply. In this case the series diverges since the nth term does not go to 0.

c) ARGUMENT: Check absolute convergence with ratio test extension: $r = \lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\frac{\ln(n+1)}{2^{n+1}} \cdot \frac{2^n}{\ln n}\right| = \lim\limits_{n\to\infty} \left|\frac{\ln(n+1)}{2\ln n}\right| = \lim\limits_{x\to\infty} \left|\frac{\ln(x+1)}{2\ln x}\right| \overset{\text{l'Ho}}{=} \lim\limits_{x\to\infty} \left|\frac{\frac{1}{x+1}}{\frac{2}{x}}\right| = \lim\limits_{x\to\infty} \left|\frac{x}{2(x+1)}\right| \overset{\text{HPwrs}}{=} \lim\limits_{x\to\infty} \left|\frac{x}{2x}\right| = \frac{1}{2} < 1$. The series converges absolutely.

d) ARGUMENT: Check absolute convergence: $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{\sqrt{2n^3+1}}\right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{2n^3+1}}$. Limit comparison with $\sum \frac{1}{n^{3/2}}$. Both series have positive terms.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\sqrt{2n^3+1}} \cdot \frac{n^{3/2}}{1} \overset{\text{HPwrs}}{=} \lim_{n\to\infty} \frac{n^{3/2}}{\sqrt{2n^3}} = \frac{1}{\sqrt{2}} > 0.$$

Since $\sum \frac{1}{n^{3/2}}$ converges (p-series, $p = \frac{3}{2} > 1$), then $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{\sqrt{2n^3+1}}\right|$ converges so $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n^3+1}}$ converges absolutely.