1. (22 points) Easy pieces to get you started. (2 points each, unless noted.)

a) Determine $\lim_{n\to\infty}\left(1-\frac{4}{n}\right)^{2n}$.

Answer

b) Determine $\int_1^\infty \frac{4}{x^{44}}\,dx$.

Answer

c) Determine whether $\sum_{n=1}^{\infty} n^{-4}$ converges. Justify your answer with a brief argument.

Answer

d) Determine $\int \cos^2(4x)\,dx$.

Answer

e) Determine $\int \sec 4x\,dx$.

Answer

f) Determine $\int 4^x + \frac{1}{4\sqrt[4]{x^3}}\,dx$.

Answer

g) Determine $\int x^3\cos(x^4+4)\,dx$.

Answer

h) Consider the function $f(x)=\frac{1}{x^2}$ on $[1,4]$ Is the left-hand Riemann sum Left(n) an overestimate or underestimate of $\int_1^4 \frac{1}{x^2}\,dx$? Explain in one sentence.

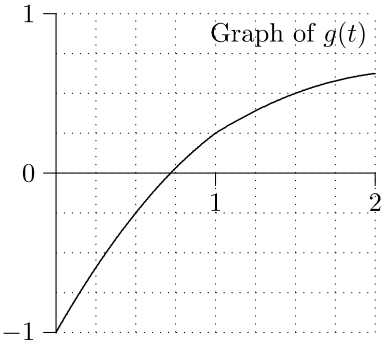
i) Find the sum of the series $\sum_{n=2}^{\infty} \frac{4}{\sqrt{k+1}} - \frac{4}{\sqrt{k}}$, or explain why it does not converge.

Answer

j) (4 points) Find the sum of the series $\sum_{n=2}^{\infty} 3\left(-\frac{4}{5}\right)^n$, or explain why it does not converge.

Answer

2. a) (4 points) Draw and then estimate the **left-hand** Riemann sum Left(4) for the graph of g on $[0, 2]$ below.



____
Answer

b) (2 points) If $g(t)$ is graphed above and $G(x) = \int_0^x g(t)\,dt$, on what interval is $G(x)$ decreasing? ____________

c) (8 points) Let $f(x) = 1 + 4x^2$ on the interval $[0, 3]$. Find and simplify the expression for the right-hand endpoint Riemann sum Right(n). Finally, evaluate $\lim_{n\to\infty}$ Right(n). How can you **check your answer**?

Answer

3. Do the following indefinite integrals. Show all work in complete detail.

a) (7 points) $\int \frac{4}{\sqrt{4-x^2}}\,dx$

Answer

b) (7 points) $\int \frac{4}{x^2\sqrt{x^2-4}}\,dx$

Answer

c) (7 points) $\int \frac{4}{\sqrt{4+x^2}}\,dx$

Answer

d) (8 points) $\int (x^2+x+1)e^{-x}\,dx$

Answer

4. a) (8 points) Calculate the arc length of $f(x) = \frac{1}{2}x^2 + 1$ on $[0, 1]$.

Answer

b) (8 points) Carefully determine $\displaystyle\int_3^5 \frac{4}{x^2 - 4x + 3}\, dx$.

Answer

c) (6 points) Carefully evaluate $\displaystyle\int_0^\infty \frac{4}{4 + x^2}\, dx$.

Answer

5. (15 points) Determine each of the following limits.

a) $\lim_{x \to 0} \frac{2x - \sin(2x)}{x^2}$

Answer

b) $\lim_{x \to 0^+} \left[1 + x^2\right]^{1/x}$

Answer

c) $\lim_{n \to \infty} \sqrt[n]{\left(1 + \frac{5}{n}\right)^{4n^2}}$

Answer

d) $\lim_{n \to \infty} 6 - 2 + \frac{2}{3} - \frac{2}{9} + \frac{2}{27} + \cdots$

Answer

e) $\lim_{n \to \infty} 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

Answer

6. **a)** (10 points) Carefully determine the **interval** of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-4)^{2n}}{4^n(2n-1)}$. Justify your answer with an argument.

Answer

b) (5 points) Carefully determine the **radius** of convergence for $\displaystyle\sum_{k=1}^{\infty} \frac{2^k k! x^k}{k^k}$. Justify your answer with an argument.

Answer

7. (10 points) Find the **AREA** enclosed by the curves $f(x) = x^3 - x^2 - 3x + 1$ and $g(x) = 5x^2 + 5x + 1$. (A graph is NOT required.)

Answer

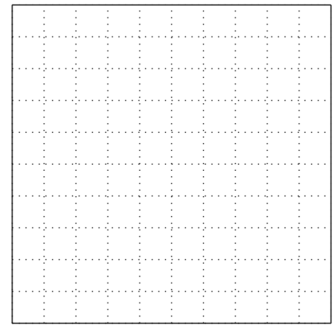
8. **a)** (10 points) Let R be the region **in the first quadrant** enclosed by the the y-axis, $y = x^2$, and $y = 3x - 2$. Rotate R about the y-axis. Determine the resulting **volume**. (Any method.)

Answer

b) (5 pts) Let R be the same region as in part (a). Rotate R about the x-axis. **Set up** the integral for the resulting **volume**. (DO NOT EVALUATE THE INTEGRAL.)

Answer

9. (10 points) Let R be the region in the first quadrant enclosed by $y = \frac{1}{4}x^2$, the y-axis, and $y = 4$. Revolve R around the y-axis to form a shallow tank. The tank has oil in it with density of 60 lbs/ft^3. If the depth of the oil is 3 feet, calculate the work done in pumping all of the oil to a height 2 feet above the top edge of the tank.



10. (9 points) Determine whether the following **arguments** are correct. Answer 'correct' if the argument is completely correct. Answer 'incorrect' if there is a mistake in the argument (indicate where the error is) even in the final answer is correct.

a) Using u-substitution, $\displaystyle\int_{-5}^{5} \frac{x}{\sqrt{x^2-9}}\,dx = \sqrt{x^2-9}\Big|_{-5}^{5} = \sqrt{16} - \sqrt{16} = 0.$

b) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n+2}$ diverges by direct comparison since $0 < \frac{1}{5n+2} < \frac{1}{n}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the p-series test.

c) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\arctan(n^2)}$ converges by the nth term test since $\displaystyle\lim_{n\to\infty} \frac{1}{\arctan(n^2)} = 0.$

11. a) (10 points) Carefully determine whether $\sum_{n=1}^{\infty} \frac{\cos(n\pi)(4n^3+1)}{n^4}$ converges absolutely, conditionally, or diverges. Justify your answer with an argument.

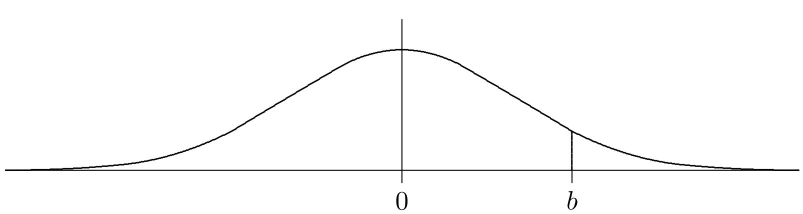
Answer

b) (5 points) Carefully determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n[(n+1)!]^3}{(3n)!}$ converges. Justify your answer with an argument.

Answer

12. (8 points) Let f be the function whose graph is given below. Use the information in the table, properties of the integral, and the **shape** of f to evaluate the given integrals.

a) $\int_3^0 f(x)\,dx$ **b)** $\int_1^4 5 + 2f(x)\,dx$ **c)** $\int_{-4}^4 f(x) + 3\,dx$ **d)** $\int_{-1}^2 f(x)\,dx$



$\int_0^1 f(x)\,dx = 0.4$
$\int_0^2 f(x)\,dx = 0.8$
$\int_0^3 f(x)\,dx = 0.9$
$\int_0^4 f(x)\,dx = 1.0$

13. a) (6 points) Determine $\int \cos^6(4x)\sin^3(4x)\,dx$

b) (8 points) Determine $\int \frac{-4x+4}{(x-2)^2 x}\,dx$

-99. Summation Formula: $\displaystyle\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

-98. Reduction Formulas for Large Powers.

1) $\displaystyle\int \cos^n x\,dx = \tfrac{1}{n}\cos^{n-1} x \sin x + \tfrac{n-1}{n}\int \cos^{n-2} x\,dx$

2) $\displaystyle\int \sin^n x\,dx = -\tfrac{1}{n}\sin^{n-1} x \cos x + \tfrac{n-1}{n}\int \sin^{n-2} x\,dx$

3) $\displaystyle\int \tan^n x\,dx = \tfrac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$

4) $\displaystyle\int \sec^n x\,dx = \tfrac{1}{n-1}\sec^{n-2} x \tan x + \tfrac{n-2}{n-1}\int \sec^{n-2} x\,dx$

-97. Guidelines for Products of Tangents and Secants:

a) If the power of secant is *even* and positive, split off $\sec^2 x$.

$$\int \tan^m x \overbrace{\sec^{2k} x}^{n=2k\text{ even}}\,dx = \int \tan^m x \overbrace{(\sec^2 x)^{k-1}}^{\text{convert to tangents}} \overbrace{\sec^2 x}^{\text{split off}}\,dx$$

b) If the power of tangent is odd and positive (and the power of secant is odd), split off $\sec x \tan x$

$$\int \overbrace{\tan^{2k+1} x}^{m=2k+1\text{ odd}} \sec^n x\,dx = \int \overbrace{(\tan^2 x)^k}^{\text{convert to secants}} \sec^{n-1} x \overbrace{\sec x \tan x}^{\text{split off}}\,dx$$

c) If m is even and n is odd, convert the tangents to secants.

$$\int \overbrace{\tan^{2k} x}^{\text{convert to secants}} \overbrace{\sec^n x}^{n\text{ odd}}\,dx$$

NAME: ______________________________

Running Total:
Practice Final Exam
Fall 2015

Problem	Points	Score
1	22	
2	14	
3	29	
4	22	
5	15	
6	15	
7	10	
8	15	
9	10	
10	9	
11	15	
12	8	
13	14	
total	198	