Application: Work

8.1 What is Work?

Work, in the physics sense, is usually defined as 'force acting over a distance.' Work is *sometimes* force times distance,[1] but not always. Work is more subtle than that. Every time you exert a force, it is not the case that any work is done (even though it may feel like that to you!

[1] Work will be force times distance in all of the applications we consider.

Why? Well, the work/energy equation says that work done (by the net force on an object) equals the object's change in kinetic energy. More simply:

$$\text{Work} = \text{Change in Kinetic Energy}. \tag{8.1}$$

This means that if an object's kinetic energy doesn't change, then no work has been done on the object—whether or not a force has been exerted. In particular, a force will do work only if the force has a component in the direction that the object moves.

Here's the distinction: if you push on a big box and it moves in the direction you push, then work is accomplished and it equals force times distance. If the box is very big and when you push it nothing happens (it does not move), then no work is done. If a force is applied to the box in a certain direction and the box moves, but not in the direction you push (maybe your friend is pushing on another side) but in some other direction, then work is done, but the amount depends on the angle the box moves relative to the direction the box moves. So again the answer is not simply force times distance.

However, to keep things simple, in the context of this section we will assume the direction the force is applied and the direction of the motion of the object are one and the same. In this case, *assuming that the force is constant*,

$$\boxed{W = \text{Work} = \text{force} \times \text{Distance} = F \cdot x.}$$

The units used to measure work vary depending on the system you are in—and they are probably less familiar to you than the units for velocity and acceleration. Table 8.1 lists the units for three common systems.

System	Force	Distance	Work
British	pounds	feet	foot-pounds (ft-lbs)
cgs	dynes	centimeters	ergs
SI (international)	Newtons	meters	joules

Table 8.1: Units of work.

EXAMPLE 8.1.1. Calculate the work done in lifting a 147 lb object 22 feet (e.g., me walking from the first to the third floor in Lansing)? Well,

$$\text{Work} = \text{force} \times \text{Distance} = 147 \times 22 = 3,234 \text{ ft-lbs}.$$

That was easy.

Now the key thing to notice is that under our assumptions, work is a *product*. One of the assumptions is that a constant force is applied. But most forces are not constant. Consider the following situation.

EXAMPLE 8.1.2. Suppose we want to hoist a leaking bucket vertically. Because of the leak, the force applied to lift the bucket decreases, so that

$$F(x) = 60\left(1 - \frac{x^2}{5000}\right), \qquad 0 \le x \le 50$$

where x is measured in feet. Find the work done in lifting the bucket.

So how do we calculate the work done if the force varies? Well, remember that Riemann sums involve products. So we will use the 'subdivide and conquer' strategy once more. Unlike in the earlier cases there is no natural figure to draw whose area, volume, or arc length we are trying to calculate. This time we must apply the theory.

General Situation. Assume that $F(x)$ is a variable but continuous force that is a function of the position x and that it is applied over an interval $[a, b]$. Find the work done. As usual, let $P = \{x_0, x_1, \ldots, x_n\}$ be a regular partition of $[a, b]$ into n equal width subintervals of length Δx. Now if the intervals are short enough, since the force is continuous, the force will be nearly constant on each interval, though its value will vary from interval to interval. So let W_i denote the work done on the ith subinterval. Since the length of the ith subinterval is Δx, then

$$W_i \approx F(x_i)\Delta x.$$

Consequently, adding up the 'pieces of work' on each subinterval,

$$\text{Total Work} = \sum_i^n W_i \approx \sum_i^n F(x_i)\Delta x. \tag{8.2}$$

Notice that we now have a Riemann sum involving the force function! To improve the approximation we do the standard thing: We let the number of subdivisions get large and take the limit. We find

$$\text{Total Work} = \lim_{n\to\infty} \sum_i^n W_i = \lim_{n\to\infty} \sum_i^n F(x_i)\Delta x = \int_a^b F(x)\,dx.$$

We are certain that this limit of the Riemann sums exists and is, in fact, the definite integral because we assumed that the force $F(x)$ is continuous function of the position.

THEOREM 8.1.3 (Work Formula). If $F(x)$ is a continuous force that is a function of the position x that is applied over an interval $[a, b]$, then the **work** done over the interval is

$$\text{Work} = \int_a^b F(x)\,dx.$$

Stop and Step Back. There are a couple of things I want you to notice. First, Theorem 8.1.3 amounts to saying that work is the 'area under the force curve.' That's probably not how you would first think of it, but that's what the theorem says!

Second, when I was writing these notes, I simply cut-and-pasted the earlier material on arc length and changed the a few words here and there. But it is the same 'subdivide and conquer' strategy that you have seen several times. You should be able to create such arguments for yourself now.

EXAMPLE 8.1.4 (Return to the leaky bucket). Return to the leaking bucket. The force applied was continuous and it was applied on the

$$F(x) = 60\left(1 - \frac{x^2}{5000}\right),$$

and it was applied on the interval $[0, 50]$ where x is measured in feet. So by Theorem 8.1.3 the work done in lifting the bucket is

$$\begin{aligned} \text{Work} = \int_a^b F(x)\,dx &= \int_0^{50} \left(1 - \frac{x^2}{5000}\right)\,dx \\ &= 60\left(x - \frac{x^3}{15,000}\right)\Bigg|_0^{50} \\ &= 60\left(50 - \frac{125,000}{15,000}\right) - 0 \\ &= 2500 \text{ ft-lbs.} \end{aligned}$$

Pretty straightforward!

8.2 Work Done Emptying a Tank

The following 'tank' problems involve pumping liquids from one height to another and determining the amount of work required to do it. Here's the general question. Given a tank containing a liquid between heights a and b, how much work is required to pump the liquid to a height H. (Note: H may or may not be the height of the top of the tank.) See Figure 8.1.

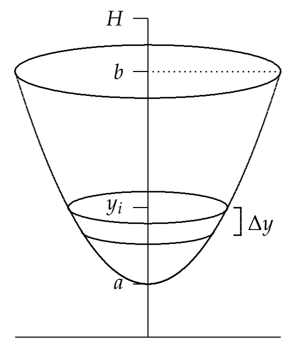

Figure 8.1: A tank containing a liquid between levels a and b. The liquid is to be moved to height H. Also shown is a representative layer of the liquid at height y_i. Can we determine how much work is required to lift this layer to height H? What will this work depend on?

To solve the problem we do the usual thing. We 'subdivide and conquer.' Use a regular partition of the interval $[a, b]$ on the vertical y-axis into n equal width subintervals, each of width Δy. The subintervals are used to create layers (slices) of liquid, each of width Δy. See Figure 8.1. What properties of the layer are important in determining the work done in lifting the layer out? Take a moment before reading on.

We need

- the volume of the layer;
- the density of the layer;
- the distance the layer is moved.

The weight of the layer is the force here (e.g., weight is measured in pounds) and is the density of the liquid times the volume. So the work to move the ith layer to height H is

$$\begin{aligned} W_i &= \text{Work to move the } i\text{th layer to height } H \\ &= \text{Weight} \times \text{Distance the layer is moved} \\ &= (\text{Density} \times \text{Volume}) \times \text{Distance the layer is moved} \end{aligned} \tag{8.3}$$

OK, it's time to use a bit of analysis and notation we saw in the section on volume.

- Let's denote the density by D (in units such as lbs/ft^3).

- As usual, let y_i denote the ith partition point of the interval $[a, b]$.
- The ith layer of liquid is approximately is approximately a cylinder (see Figure 8.1) so its volume is

$$V_i = \text{Area of the base} \times \text{height} = \text{Cross-sectional area} \times \text{height} = A(y_i)\Delta y,$$

  where $A(y_i)$ denotes the the cross-sectional area of the tank at height y_i and the height of the layer is Δy.
- Finally, since the ith layer is at height y_i and has to be moved to height H, then the ith layer is moved a distance of $H - y_i$.

Putting this all together, we can rewrite the work to move the ith layer to height H in (8.4) as

$$\begin{aligned} W_i &= (\text{Density} \times \text{Volume}) \times \text{Distance the layer is moved} \\ &= D \times V_i \times (H - y_i) \\ &\approx D[A(y_i)\Delta y](H - y_i). \end{aligned} \tag{8.4}$$

But now you know how the rest of the story goes. To estimate the total work we add up the work to move each layer for each subinterval,

$$\text{Total Work} = \sum_i^n W_i \approx \sum_i^n D[A(y_i)\Delta y](H - y_i). \tag{8.5}$$

We now have a Riemann sum.[2]

To improve the approximation we do the standard thing: We let the number of subdivisions get large and take the limit. We find

$$\text{Total Work} = \lim_{n\to\infty} \sum_i^n W_i = \lim_{n\to\infty} \sum_i^n D[A(y_i)\Delta y](H - y_i) = D \int_a^b A(y)[H - y]\, dy.$$

We are certain that this limit of the Riemann sums exists and is, in fact, the definite integral as long as we assume that the cross-sectional area $A(y)$ is continuous function of y.

[2] Take a second to compare (8.2) and (8.5). Then take another to anticipate the rest of this argument.

THEOREM 8.2.1 (Work Formula for Emptying a Tank). Assume the cross-sectional $A(y)$ is a continuous function of the position y and that the density of the contents is a constant D. If the contents of the tank to be moved lie in the interval $[a, b]$, then the **work** done to move this material to a height H is

$$\text{Work} = D \int_a^b A(y)[H - y]\, dy.$$

Caution: The tank may not be full, the contents may be moved to a height H above the tank, or the entire tank may not be emptied. If the tank is being *filled* from a source at height H (either at the bottom of or below the tank), then the contents must be moved to each layer height y between a and b so the distance moved is $y - H$ rather than $H - y$.

EXAMPLE 8.2.2. Here's a simple first example. An above-ground backyard swimming pool has the shape of a circular cylinder with a radius of 10 ft and a depth of 8 ft. Assume the depth of the water in the pool is 5 ft. Find the work done in emptying the pool by pumping the water over the top edge of the pool. Note: The density of water is 62.5 lbs/ft^3. See Figure 8.2.

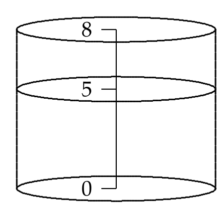

Figure 8.2: The pool is filled to a depth of 5 feet and the water is to be pumped out over the top edge at height 8 feet.

Solution. We apply Theorem 8.2.1. Be attentive to the different heights. The liquid lies between 0 and 5 feet but has to be moved to a height of $H = 8$ feet. The cross-sections are circles of radius $r = 10$ feet. So $A(y) = \pi r^2 = 100\pi$. (In most problems the cross-sections will vary.) So

$$\begin{aligned}
\text{Work} = D\int_a^b A(y)[H-y]\,dy = 62.5\int_0^5 100\pi[8-y]\,dy &= 6250\pi\left(8y-\frac{y^2}{2}\right)\Bigg|_0^5 \\
&= 6250\pi\left[\left(40-\frac{25}{2}\right)-0\right] \\
&= 17,1875\pi \text{ ft-lbs.}
\end{aligned}$$

EXAMPLE 8.2.3. (A revision) Suppose the swimming pool in Example 8.2.2 above was *in-ground* so that the top of the pool was at ground level. How would the work integral change? Would the work done emptying the pool change?

Solution. Three things change in the set-up. The liquid now lies between -8 and -3 feet and has to be moved to a height of $H = 0$ feet. The cross-sections are the same.

$$\begin{aligned}
\text{Work} = D\int_a^b A(y)[H-y]\,dy = 62.5\int_{-8}^{-3} 100\pi[0-y]\,dy &= 6250\pi\left(-\frac{y^2}{2}\right)\Bigg|_{-8}^{-3} \\
&= 6250\pi\left[-\frac{9}{2}+32\right] \\
&= 17,1875\pi \text{ ft-lbs.}
\end{aligned}$$

The work should and does remain the same.

EXAMPLE 8.2.4. (A tank formed by rotation) A tank is formed by rotating the region between $y = x^2$, the y-axis and the line $y = 4$ in the first quadrant around the y axis. The tank is filled with oil with density 50 lbs/ft^3.

(*a*) Find the work done pumping the oil to the top of the tank.

(*b*) Find the work done pumping the oil to the top of the tank if there is only 1 foot of oil in the tank.

(*c*) Suppose the tank is empty and is **filled** from a hole in the bottom to a depth of 3 feet. Find the work done.

Solution. Figure 8.3 shows a sketch of the tank with a representative cross-section. The circular cross-section formed by rotating the point (x, y) about the y-axis has radius $r = x$. So the cross-sectional area is

$$A(y) = \pi r^2 = \pi x^2 = \pi y,$$

as a function of y.

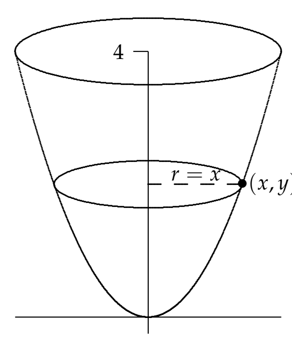

Figure 8.3: The circular cross-section formed by rotating the point (x, y) about the y-axis has radius $r = x$.

(*a*) We apply Theorem 8.2.1.

$$\begin{aligned}
\text{Work} = D\int_a^b A(y)[H-y]\,dy &= 50\int_0^4 \pi(y)[4-y]\,dy \\
&= 50\int_0^4 \pi\left(4y-y^2\right)\,dy \\
&= 50\pi\left(2y^2-\frac{y^3}{3}\right)\Big|_0^4 \\
&= 50\pi\left[\left(32-\frac{64}{3}\right)-0\right] \\
&= \frac{1600\pi}{3}\text{ ft-lbs.}
\end{aligned}$$

(*b*) If there is only 1 foot of oil in the tank, the only change is that the upper limit of integration is now 1 instead of 4. (Remember the limits of integration represent the upper an lower levels of the content of the tank.) So

$$\begin{aligned}
\text{Work} = D\int_a^b A(y)[H-y]\,dy &= 50\int_0^1 \pi(y)[4-y]\,dy \\
&= 50\pi\left(2y^2-\frac{y^3}{3}\right)\Big|_0^1 \\
&= 50\pi\left[\left(2-\frac{1}{3}\right)\right] \\
&= \frac{250\pi}{3}\text{ ft-lbs.}
\end{aligned}$$

(*c*) The key observation when a tank is **filled from the bottom** is that each layer must be 'moved' or lifted from ground level 0 to its final height $H = y$. Since the tank is only filled to a depth of 3 feet, the limits of integration are 0 (bottom) to 3 ft. So this time

$$\begin{aligned}
\text{Work} = D\int_a^b A(y)[H-y]\,dy &= 50\int_0^3 \pi(y)[y-0]\,dy \\
&= 50\pi\left(\frac{y^3}{3}\right)\Big|_0^3 \\
&= 50\pi\left[\left(2-\frac{1}{3}\right)\right] \\
&= 450\pi\text{ ft-lbs.}
\end{aligned}$$

YOU TRY IT 8.1. How would the integral and work change if the tank were full and the oil was pumped to a height 3 feet above the top of the tank? (Answer: $\frac{5200}{3}\pi$ ft-lbs.)

EXAMPLE 8.2.5. (A more complicated problem) An underground hemispherical tank with radius 10 ft is filled with oil of density 50 lbs/ft^3. Find the work done pumping the oil to the surface if the top of the tank is 6 feet below ground.

Solution. It will be easiest to set up the equation of the hemisphere if we think of the top of the tank at height 0 and then pump the oil to a height of 6 feet. See Figure 8.4.

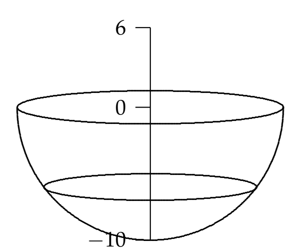

Figure 8.4: The tank with its position re-imagined and a representative layer.

The cross-sections are circles. We will be able to determine the cross-sectional area once we determine the radius of the cross-section. The semi-circle is part of the circle of radius 10 centered at the origin which has equation $x^2 + y^2 = 10$. The radius of a cross-section is the x-coordinate of the point (x, y) that lies on the semi-circle in fourth quadrant. (See Figure 8.5.) Thus,

$$r = x = \sqrt{(10)^2 - y^2}.$$

Therefore the cross-sectional area is

$$A(y) = \pi r^2 = \pi[10^2 - y^2] = \pi(100 - y^2).$$

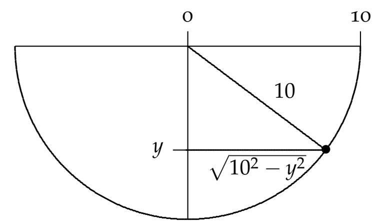

Figure 8.5: The radius of the cross-section at height y is $\sqrt{(10)^2 - y^2}$.

We apply Theorem 8.2.1. Remember the liquid is pumped to height $H = 6$ in our re-casting of the problem.

$$\begin{aligned}
\text{Work} = D\int_a^b A(y)[H-y]\,dy &= 50\int_{-10}^{0} \pi((10)^2 - y^2)[6-y]\,dy \\
&= 50\int_{-10}^{0} \pi\left(600 - 100y - 6y^2 + y^3\right)\,dy \\
&= 50\pi\left(600y - 50y^2 - 2y^3 + \frac{y^4}{4}\right)\Bigg|_{-10}^{0} \\
&= 50\pi\left[0 - (-6000 - 5000 + 2000 + 2500)\right] \\
&= 325{,}000\pi \text{ ft-lbs.}
\end{aligned}$$

YOU TRY IT 8.2. Set up the new integral for each modification of the example above and determine the work required.

(*a*) How would the integral and work change if the tank were only 5 feet below ground? (Answer: $\frac{875000}{3}\pi$ ft-lbs.)

(*b*) How would the integral and work change if the top of the tank were at ground level? (Answer: $125{,}000\pi$ ft-lbs.)

YOU TRY IT 8.3. Suppose that a tank is formed by rotating the region in the first quadrant enclosed by $y = (x+2)^2$, the y-axis, and the x-axis about the y-axis (see Figure 8.6).

(*a*) Find the work done pumping the oil to the top of the tank. (Answer: $50\int_0^4 \pi(\sqrt{y} - 2)^2(4-y)\,dy = \frac{1280}{3}\pi$ ft-lbs.)

(*b*) Find the work done if the oil was pumped to a height 3 feet above the top of the tank? (Answer: $\frac{2480}{3}\pi$ ft-lbs.)

(*c*) Find the work done pumping the oil to the top of the tank if there is only 1 foot of oil in the tank. (Answer: 330π ft-lbs.)

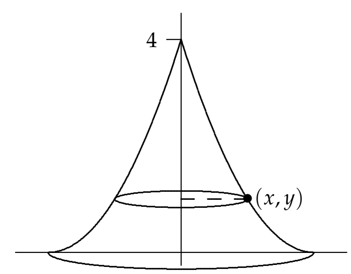

Figure 8.6: The circular cross-section formed by rotating the point (x, y) about the y-axis has radius $r = x = \sqrt{y} - 2$.

EXAMPLE 8.2.6. (A more complicated problem) A large gasoline storage tank lies 3 feet underground. It is in the shape of a half-cylinder (flat side up) 4 ft in radius and 8 ft long. If the tank is full of gas with density 60 lbs/ft^3, find the work done pumping the gas to a level 2 ft above ground.

Solution. It will be easiest to set up the equation of the hemisphere if we think of the top of the tank at height 0. Since the original tank was 3 feet *underground* and the contents were to be pumped to 2 feet *above* ground, in our model we need to pump the gas to a total of 5 feet above ground. See Figure 8.7.

The cross-sections are rectangles of width $2x$ and length 8. Since $x^2 + y^2 = 4$ on the semi-circle, $x = \sqrt{16 - y^2}$. So the cross-sectional area is

$$A(y) = 2x(8) = 16\sqrt{16 - y^2}.$$

We apply Theorem 8.2.1. Remember the liquid is pumped to height $H = 5$ in our re-casting of the problem.

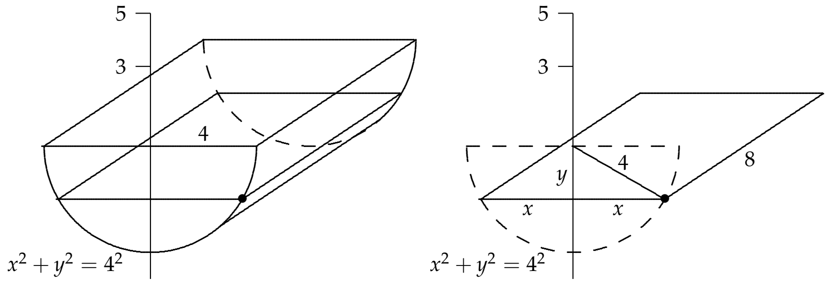

Figure 8.7: Left: Think of the tank top at ground level and the liquid being pumped to height $H = 5$ ft. Right: The width of the cross-section at height y is $2x = 2\sqrt{4^2 - y^2}$.

$$\begin{aligned}\text{Work} = D\int_a^b A(y)[H-y]\,dy &= 50\int_{-4}^{0}(16\sqrt{16-y^2}[5-y]\,dy \\ &= 800\int_{-4}^{0}\left(5\sqrt{16-y^2} - y\sqrt{16-y^2}\right)\,dy \\ &= 4000\int_{-4}^{0}\sqrt{16-y^2}\,dy - 800\int_{-4}^{0} y\sqrt{16-y^2}\,dy.\end{aligned} \tag{8.6}$$

The first integral $\int_{-4}^{0}\sqrt{16-y^2}\,dy$ represents the area of a quarter circle of radius $r = 4$ and so its value[3] (area) is $\frac{4^2\pi}{4} = 4\pi$.

The second integral in (8.6) can be done using the substitution $u = 16 - y^2$ with $-\frac{1}{2}\,du = y\,dy$. Be careful to change the limits!

$$\int_{-4}^{0} y\sqrt{16-y^2}\,dy = \int_0^{16}\frac{1}{2}u^{1/2}\,du = \frac{1}{3}u^{3/2}\Big|_0^{16} = \frac{64}{3}.$$

Consequently, (8.6) becomes

$$\begin{aligned}\text{Work} &= 4000\int_{-4}^{0}\sqrt{16-y^2}\,dy - 800\int_{-4}^{0} y\sqrt{16-y^2}\,dy \\ &= 4000(4\pi) - 800\pi\left(\frac{64}{3}\right) \\ &\approx 33172.14913 \text{ ft-lbs.}\end{aligned}$$

[3] Remember that we have not yet developed an antiderivative of $\sqrt{16-y^2}$, as we noted in the section on arc length, so we have to use the **geometric interpretation** of the definite integral here.

YOU TRY IT 8.4. How would the integral and work change if the tank in the previous problem had the flat side facing down? Hint: Put the center of the semi-circle at the origin. The tank now lies *above* the x-axis. What height is the gas pumped to this time? Now work out the answer. (Answer: $28800\pi - \frac{51200}{3}$ ft-lbs.)

YOU TRY IT 8.5. A tank in the form of a truncated cone is formed by rotating the segment between $(2, 0)$ and $(4, 4)$ around the y-axis. It is filled with sludge (density 80 lbs/ft^3). If the sludge is pumped 3 feet upwards into a tank truck, how much work was required? (Answer: $\frac{40960\pi}{3}$ ft-lbs.)

YOU TRY IT 8.6 (Rotation about a non-axis line). This time a tank in the form of a truncated cone is formed by rotating the segment between $(0, 4)$ and $(2, 0)$ around the line $x = 4$. It is filled with sludge (density 80 lbs/ft^3). If the sludge is pumped 3 feet upwards into a tank truck, how much work was required? (Answer: $\frac{40960\pi}{3}$ ft-lbs.)

YOU TRY IT 8.7 (Non-rotation). A triangular trough for cattle is 8 ft long. The ends are triangles with with a base of 3 ft and height of 2 ft (but the vertex points down). See Figure 8.8. Find the work done by the cattle in emptying just the top foot of water (density 62.5 lbs/ft^3) over the edge. (Answer: 500 ft-lbs.)

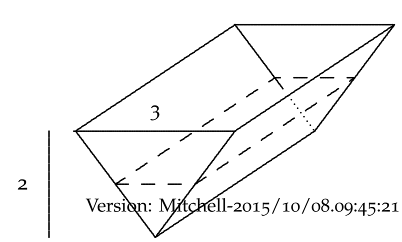

Figure 8.8: Find the work done in emptying the top foot of the trough

YOU TRY IT 8.8. A cup shaped tank is obtained by rotating the curve $y = x^3$ about the y-axis where $0 \leq x \leq 2$.

(*a*) Assume the tank is full of water (density 62.5 lbs/ft^3?). How much work is done in emptying the tank by removing the water over the top edge of the tank? (Answer: 3600π ft-lbs?)

(*b*) How much work would be done in raising the water 2 feet above the tank's top?

(*c*) Suppose the depth of the liquid in the tank is 1 foot. Find the work required to pump the liquid to the top edge of the tank.

YOU TRY IT 8.9. (*a*) A cone-shaped reservoir has a 10 foot radius across the top and a 15 foot depth. If the reservoir has 9 feet of oil (density 54 lbs/ft^3) in it, how much work is required to empty it by bringing the water to the top of the reservoir? (First figure out the equation of the line that determines the cone.)

(*b*) Same question with the reservoir being completely full. (Answer: 101250π ft-lbs?)

YOU TRY IT 8.10 (Expanding Gas). Boyle's law says that the pressure (force) exerted by a gas is inversely proportional to the volume. A quantity of gas with an initial volume of 2 cubic feet and a pressure of 1000 pounds per square foot expands to a volume of 5 cubic feet. Find the work done. Hint: $p = k/V$, so first solve for k. Next, the work done by an expanding gas is

$$W = \int_{V_0}^{V_1} p\,dv.$$

Answer: $2000(\ln 5 - \ln 2)$ ft. lbs.)

YOU TRY IT 8.11 (Extra Credit). A heavy rope is 60 feet long and has a density of 1.5 lbs/ft. It is hanging over the edge of a building 100 ft tall. Find the work done in pulling the rope to the top of the building. Hint: Model your answer like a tank problem. Think of the rope in sections. What force must be applied to move each section of the rope? What distance must each section be used? OK, set up the integral and do it!

8.3 Problems

1. Assume 25 ft-lb of work is required to stretch a spring 3 feet beyond its natural length. Find the work done in stretching the spring from 2 to 4 ft beyond its natural length. Hint: First find the spring constant. (Answer: $k = 50/9$ lbs/ft and 100/3 ft-lbs.)

2. Let R be the region in the first quadrant bounded by the x-axis, the y-axis, and the curve $y = 4 - x^2$. Rotate the R around the y-axis to form a silo tank. If the tank is filled with wheat (100 lbs per cu. ft) how much work is done in raising the wheat to the top of the silo. (Ans: $\frac{6400}{3}\pi$ ft lbs.)

3. Let R be the region in the first quadrant bounded by the y-axis and the curves $y = x^2$ and $y = 6 - x$. Rotate the R around the y-axis to form a tank. If the tank is filled with honey (90 lbs per cu. ft) how much work is done in raising the honey to the top of the tank. Note: you will have to divide the work into two pieces! Be careful of the limits. (Ans: 2760π ft lbs.)

4. A beer ball with radius 1 foot is located with its center at the origin. It has a tap 0.5 ft long sticking up out the top. If beer has a density of 60 pounds per cubic foot, find the work done in emptying the beer ball. Hint: Locate the center of the ball at the origin. (Ans: 120π ft-lbs)

5. Ants excavate a chamber underground that is described as follows: Let S be the region in the fourth quadrant enclosed by $y = -\sqrt{x}$, $y = -1$, and the y-axis; revolve S around the y-axis.

   (*a*) Set up and simplify the integral for the volume of the chamber using the shell method. (For future reference: Ans: $\pi/5$)

(*b*) [From an exam] Suppose that the chamber contained soil which weighed 50 lbs per cubic foot. How much work did the ants do in raising the soil to ground level? (Ans: $50\pi/6$ ft. lbs.)

If you have time, set up each of the following integrals which we will do with Maple.

6. A small farm elevated water tank is in the shape obtained from rotating the region in the first quadrant enclosed by the curves $y = 10 - \frac{1}{2}x^2$, $y = 8$, and the y-axis about the y-axis.

(*a*) Find the work "lost" if the water (62.5 lbs/ft^3) leaks onto the ground from a hole in the bottom of the tank. (Answer: $-6500\pi/3$ ft-lbs.)

(*b*) Find the work "lost" if the water leaks onto the ground from a hole in the side of the tank at height 9 feet. (Answer: $-1750\pi/3$ ft-lbs.)

7. The line segment between the points $(1, -3)$ and $(3, -1)$ is rotated around the y-axis to form a truncated conical gas storage tank. If the tank has only 1 foot of gasoline (density 60 lbs/ft^3) in it, **set up the integral** for the work done to pump all of the gas to ground level.

WEBWORK: Click to try Problems 95 through 98. Use GUEST login, if not in my course.