Math 131 Day 08

#1) a We want the change in volume $V(3)-V(1)$

$$V(3)-V(1) = \int_1^3 V'(t)\,dt = \int_1^3 \frac{1}{3t+1}\,dt = \frac{1}{3}\int_1^3 \frac{3}{3t+1}\,dt$$

problems 6, 7 on lab

"Net Change" $= \frac{1}{3}\ln|3t+1|\Big|_1^3 = \frac{1}{3}\left[\ln 10 - \ln 4\right] = \frac{1}{3}\ln 10/4$

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$$= \boxed{\frac{1}{3}\ln 5/2} \quad (\approx 0.3054)$$

#2) $y=F(x)=\int_{x^4}^2 8\sin(\pi t^2)\,dt$. So

$u = x^4 \qquad \frac{du}{dx} = 4x^3$

$$\frac{dy}{dx} = F'(x) = \frac{d}{dx}\left[\int_{x^4}^2 8\sin(\pi t^2)\,dt\right] = \frac{d}{dx}\left[-\int_2^{x^4} 8\sin(\pi t^2)\,dt\right]$$

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$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{d}{du}\left[-\int_2^{u} 8\sin(\pi t^2)\,dt\right]\frac{du}{dx}$$

$u^2 = (x^4)^2 = x^8$

$$= -8\sin(\pi u^2)\cdot 4x^3 = -8\sin(\pi x^8)\cdot 4x^3$$
$$= \boxed{-32x^3\sin(\pi x^8)}$$

#3 $\int_{1/2}^{x} g(t)\,dt = x^2\ln x$. Find $g(1)$. Take derivative of both sides to get $g(x)$: $\frac{d}{dx}\left[\int_1^x g(t)\,dt\right] = \frac{d}{dx}\left(x^2\ln x\right)$

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So $g(x) = 2x\ln x + x^2\cdot\frac{1}{x}$

$g(x) = 2x\ln x + x$

So $g(1) = 2\cdot\ln 1 + 1 = \boxed{1}$

#4 $f_{ave} = \frac{1}{b-a}\int_a^b f(t)\,dt = \frac{1}{4-2}\int_2^4 \sin\left(\frac{\pi}{2}t\right)dt = -\frac{1}{2}\cdot\frac{2}{\pi}\cos\left(\frac{\pi}{2}t\right)\Big|_2^4$

4)

$$= -\frac{1}{\pi}\left[\cos 2\pi - \cos\pi\right] = -\frac{1}{\pi}\left[1-(-1)\right] = -\frac{2}{\pi}$$

XC: Net change $= \int_2^4 \sin\left(\frac{\pi}{2}t\right)dt = -\frac{2}{\pi}\cos\left(\frac{\pi}{2}t\right)\Big|_2^4 = -\frac{2}{\pi}\left[1-(-1)\right] = -\frac{4}{\pi}$

"Exhaling" (neg change)

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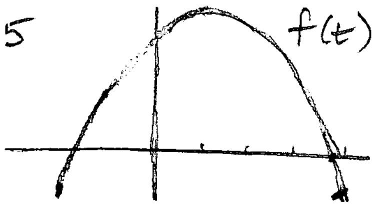
\#5  $f(t) = A'(t)$

a) loc max where $A'(t) = f(t)$ changes from pos to neg: At $t = 3.5$

b) $A(t)$ is increasing where $A'(t) = f(t)$ is positive: $(-1.5, 3.5)$

4) c) $A(4)$ is positive. The net area from -2 to 4 is clearly positive

d) $B(0) = \int_3^0 f(t)\,dt = -\int_0^3 f(t)\,dt$ is negative.

The region is above the axis — but we multiply by -1 (reversed)

\# 6) $\int_{-101}^{101} x^9 - 5x^3 - 4x\,dx = 0$ because the function is odd and the interval is symmetric about 0

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\# 7) $f_{ave} = \frac{1}{b-a}\int_a^b f(t)\,dt = \frac{1}{4-1}\int_1^4 1/x\,dx = \frac{1}{3}\ln|x|\Big|_1^4 = \boxed{\frac{1}{3}\ln 4}$

4) we need c betw 1 and 4 so that

$f(c) = \frac{1}{c} = \frac{1}{3}\ln 4$. So $c = \frac{3}{\ln 4} \approx 2.164$

\#8 $\frac{d}{dx}\left[\int_1^x \ln(t^2+1)\,dt + \int_x^{100} \ln(t^2+1)\,dt\right]$

2) $= \frac{d}{dx}\left[\int_1^x \ln(t^2+1)\,dt - \int_{100}^{x} \ln(t^2+1)\,dt\right]$

$= \ln(x^2+1) - \ln(x^2+1) = \boxed{0}$