Day 11 Math 131

#1 $v(t) = t^3 - 4t^2 + 3t$ m/s on $[0,4]$

a) Moving forward & backward

$$v(t) = t^3 - 4t^2 + 3t = t(t^2 - 4t + 3) = t(t-1)(t-3) = 0$$

at $t = 0, 1, 3$

$v(t)$	0	++	0	− − −	0	+ +++
	0		1		3	4

forward on $[0,1]$ and $[3,4]$

backward on $[1,3]$

b) displacement $= \int_0^4 t^3 - 4t^2 + 3t\,dt = \frac{t^4}{4} - \frac{4t^3}{3} + \frac{3t^2}{2}\Big|_0^4$

$$= 64 - \frac{256}{3} + 24 - 0 = \boxed{\frac{8}{3}} \text{ m}$$

c) $v_{ave} = \frac{1}{4-0}\int_0^4 t^3 - 4t^2 + 3t\,dt = \frac{1}{4}\left(\frac{8}{3}\right) = \frac{2}{3}$ m/s

d) Dist travelled $= \int_0^3 |t^3 - 4t^2 + 3t|\,dt =$

moving backwards

$$= \int_0^1 t^3 - 4t^2 + 3t\,dt + \int_1^3 -(t^3 - 4t^2 + 3t)\,dt$$

$$= \frac{t^4}{4} - \frac{4t^3}{3} + \frac{3t^2}{2}\Big|_0^1 - \left(\frac{t^4}{4} - \frac{4t^3}{3} + \frac{3t^2}{2}\right)\Big|_1^3$$

$$= \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2} - 0\right) - \left[\left(\frac{81}{4} - \frac{108}{3} + \frac{27}{2}\right) - \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2}\right)\right]$$

$$= -\frac{79}{4} + \frac{100}{3} - \frac{21}{2} = \frac{-237 + 400 - 126}{12} = \frac{37}{12} \approx 3.08\overline{3} \text{ m}$$

page 408 #22: $a(t) = e^{-t}$, $v(0) = 60$, $s(0) = 40$

$$v(t) = v(0) + \int_0^t e^{-x}\,dx = 60 + \left[-e^{-x}\right]_0^t = 60 - e^{-t} + 1 = 61 - e^{-t}$$

$$s(t) = s(0) + \int_0^t 61 - e^{-x}\,dx = 40 + \left[61x + e^{-x}\right]_0^t$$

$$= 40 + [61t + e^{-t} - 1]$$

$$= \boxed{39 + 61t + e^{-t}}$$

Remember $v(t) = v(0) + \int_0^t a(x)\,dx$ use $v(t)$ here

$$s(t) = s(0) + \int_0^t v(t)\,dt$$

p 408 #32 $\quad a(t) = \dfrac{20}{(t+2)^2}, \quad v(0) = 20, \quad s(0) = 10$

$u = x+2$
$du = dx$

$$v(t) = v_0 + \int_0^t a(x)\,dx = 20 + \int_0^t \frac{20}{(x+2)^2}\,dx = 20 + \int_0^t 20(x+2)^{-2}\,dx$$

$$= 20 - \left[20(x+2)^{-1}\Big|_0^t\right] = 20 - \left[\frac{20}{t+2} - \frac{20}{2}\right] = \boxed{30 - \frac{20}{t+2}}$$

$$s(t) = s(0) + \int_0^t v(x)\,dx = 10 + \int_0^t 30 - \frac{20}{x+2}\,dx$$

$$= 10 + \Big[30x - 20\ln(x+2)\Big]\Big|_0^t$$

$$= 10 + \big(30t - 20\ln(t+2) - (0 - 20\ln(2))\big)$$

$$= 10 + 20\ln(2) + 30t - 20\ln(t+2)$$

p 408 #40 (see bottom of page)

$P'(t) = 20 - t/5 \qquad P(0) = 55$

$$P(6) = P(0) + \int_0^6 P'(t)\,dt = 55 + \int_0^6 20 - \frac{t}{5}\,dt = 55 + \left[20t - \frac{t^2}{10}\right]_0^6$$

$$= 55 + [120 - 36/10 - 0] = 171.4$$

$$P(t) = 55 + \int_0^t P'(x)\,dx = 55 + \int_0^t 20 - \frac{x}{5}\,dx = 55 + \left[20x - \frac{x^2}{10}\right]_0^t$$

$$= 55 + \left[20t - \frac{t^2}{10} - 0\right]$$

$$P(t) = 55 + 20t - \frac{t^2}{10}$$

p410 #60

a) $Q(1) = \overset{\text{empty}}{Q(0)} + \displaystyle\int_0^{60} 3\sqrt{t}\,dt = 0 + 2t^{3/2}\Big|_0^{60 \text{ minutes}} = 2(60)^{3/2} \approx \boxed{929.51 \text{ liters}}$

b) $Q(t) = Q(0) + \displaystyle\int_0^t 3\sqrt{x}\,dx = 0 + 2x^{3/2}\Big|_0^t = \boxed{2t^{3/2}}$

Remember: $\displaystyle\int_0^b P'(t)\,dt = P(b) - P(0)$.

This is why $P(0)$ is here

Solving to $P(b)$ we get $P(b) = P(0) + \displaystyle\int_0^b P'(t)\,dt$

The same is true for: $P(t) = P(0) + \displaystyle\int_0^t P'(x)\,dx$