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Math 131 Day 12

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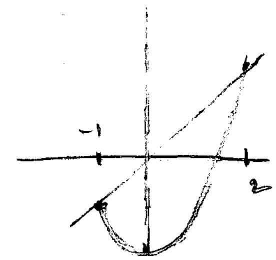
#1 Region enclosed by $y = x$ and $y = x^2 - 2$

Intersect: $x = x^2 - 2 \Rightarrow x^2 - x - 2 = (x+1)(x-2) = 0$

$x = -1, 2$



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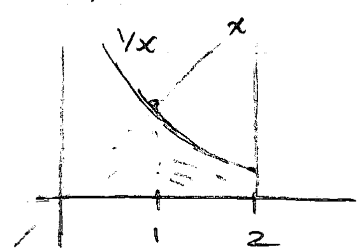
$$\text{Area} = \int_{-1}^{2} x - (x^2 - 2)\,dx = \frac{x^2}{2} - \frac{x^3}{3} + 2x \Big|_{-1}^{2}$$

$$= 2 - 8/3 + 4 - (1/2 + 1/3 - 2) = 4\tfrac{1}{2}$$

#17 Enclosed by $y = x$, $y = 1/x$, $y = 0$, $x = 2$

Intersect $x = 1/x \Rightarrow x^2 = 1 \Rightarrow x = 1, \cancel{-1}$



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$$\text{Area} = \int_0^1 x\,dx + \int_1^2 1/x\,dx$$

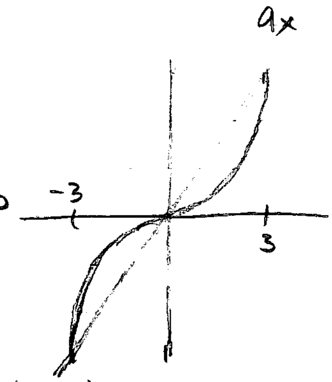
$$= \frac{x^2}{2}\Big|_0^1 + \ln|x|\Big|_1^2 = (1/2 - 0) + (\ln 2 - \ln 1) = 1/2 + \ln 2$$

#20 Enclosed by $y = x^3$ and $y = 9x$

Intersect: $x^3 = 9x \Rightarrow x^3 - 9x = x(x^2 - 9)$

$= x(x-3)(x+3) = 0$

$x = -3, 0, 3$



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$$\text{Area} = \int_{-3}^{0} x^3 - 9x\,dx + \int_0^3 9x - x^3\,dx$$

symmetry (odd ... but doubles)

$$= 2\int_0^3 9x - x^3\,dx$$

$$= 2\left[\frac{9}{2}x^2 - \frac{x^4}{4}\Big|_0^3\right] = 2\left(\left(\frac{81}{2} - \frac{81}{4}\right) - 0\right) = \frac{81}{2}$$

#22 Enclosed by $y = x^2(3-x)$ and $y = 12 - 4x$

Intersect:

$x^2(3-x) = 12 - 4x = 4(3-x) \Rightarrow x^2(3-x) = 4(3-x)$

so either $\boxed{x = 3}$ or $x^2 = 4 \Rightarrow \boxed{x = \pm 2}$

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Which curve is on top? $\quad x^2(3-x) \quad\quad 12 - 4x$

on $[-2, 2]$ ... at $x = 0$: $0^2(3-0) = 0$ AND $12 - 4(0) = \boxed{12}$

on $[2, 3]$ at 2.5: $(2.5)^2(.5) = \boxed{3.125}$ $\quad 12 - 10 = 2$

$$\text{Area} = \int_{-2}^{2} 12 - 4x - x^2(3-x)\,dx + \int_2^3 x^2(3-x) - (12 - 4x)\,dx$$

odd, even

$$= \int_{-2}^{2} 12 - 4x - 3x^2 + x^3\,dx + \int_2^3 3x^2 - x^3 - 12 + 4x\,dx$$

$$= 2\int_0^2 12 - 3x^2\,dx + \left(x^3 - \frac{x^4}{4} - 12x + 2x^2\right)\Big|_2^3 =$$

$$= 2(12x - x^3)\Big|_0^2 + (27 - 81/4 - 36 + 18) - (8 - 4 - 24 + 8) = \boxed{32} + \boxed{3/4}$$

$= 32\tfrac{3}{4} = 131/4$

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\# 16 The two curves meet at 0 and π (obvious) and at $\pi/3$ because $\sin\frac{\pi}{3} = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$.

Using the graph shown

top top

$$\text{Area betw} = \int_0^{\pi/3} \sin 2x - \sin x\,dx + \int_{\pi/3}^{\pi} \sin x - \sin 2x\,dx$$

$$= -\tfrac{1}{2}\cos 2x + \cos x \Big|_0^{\pi/3} + \left(-\cos x + \tfrac{1}{2}\cos 2x\right)\Big|_{\pi/3}^{\pi}$$

(5)

$$= \left(-\tfrac{1}{2}\cos\tfrac{2\pi}{3} + \cos\tfrac{\pi}{3}\right) - \left(-\tfrac{1}{2}+1\right) + \left(-(-1) + \tfrac{1}{2}(1)\right) - \left(-\cos\tfrac{\pi}{3} + \tfrac{1}{2}\cos\tfrac{2\pi}{3}\right)$$

$1/2 \quad -1/2$

$$= 2\cos\tfrac{\pi}{3} - \cos\tfrac{2\pi}{3} - 1/2 + 3/2$$

$$= 1 + 1/2 - 1/2 + 3/2 = 5/2$$

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\# 7 The two curves meet at $x=1$ because $2^1 = 3-1$

$$\text{Area} = \int_0^1 (3-x) - 2^x\,dx = 3x - \frac{x^2}{2} - \frac{2^x}{\ln 2}\Big|_0^1$$

(3)

$$= \left(3 - 1/2 - \frac{2}{\ln 2}\right) - \left(0 - 0 - \frac{1}{\ln 2}\right)$$

$$= 2\,1/2 - \frac{1}{\ln 2}$$

\# 7

(2) $\text{Shaded Area} = \int_0^{2.5} h(x) - f(x)\,dx + \int_{2.5}^{4.5} g(x) - f(x)\,dx$

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#42 $P'(t) = 5 + 10\sin\frac{\pi t}{5}$, $P(0) = 35$

$$P(15) = P(0) + \int_0^{15} 5 + 10\sin\frac{\pi t}{5}\,dt = 35 + \left[5t - \frac{50}{\pi}\cos\frac{\pi t}{5}\right]_0^{15}$$

$$= 35 + \left[\left(75 - \frac{50}{\pi}\cos 3\pi\right) - \left(0 - \frac{50}{\pi}\cos 0\right)\right]$$

As above

$$= 35 + \left(75 + \frac{50}{\pi}\right) - \left(-\frac{50}{\pi}\right) = 110 + \frac{100}{\pi} \approx 141.83$$

$$P(35) = 35 + \left[5t - \frac{50}{\pi}\cos\frac{\pi t}{5}\right]_0^{35} = 35 + \left(175 + \frac{50}{\pi}\right) + \left(\frac{50}{\pi}\right)$$

$$= 210 + \frac{100}{\pi} \approx 241.83$$

$$P(t) = P(0) + \int_0^t 5 + 10\sin\frac{\pi x}{5}\,dx = 35 + \left[5x - \frac{50}{\pi}\cos\frac{\pi x}{5}\right]_0^t$$

$$= 35 + \left[5t - \frac{50}{\pi}\cos\frac{\pi t}{5} - \left(-\frac{50}{\pi}\right)\right]$$

$$= 5t - \frac{50}{\pi}\cos\frac{\pi t}{5} + 35 + \frac{50}{\pi}$$