Math 131 Day 18

we don't need to find this

#1 Segment betw $(2,4)$ & $(5,13)$ ... $f(x) = mx + b$ --- we need

$$f'(x) = m = \text{slope} = \frac{13-4}{5-2} = 3$$

$$AL = \int_2^5 \sqrt{1+(3)^2}\, dx = \sqrt{10}\, x \Big|_2^5 = \sqrt{10}(5-2) = \boxed{3\sqrt{10}}$$

#2 $f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ on $[1,e]$. $f'(x) = \frac{1}{2}x - \frac{1}{2}x^{-1}$

$$AL = \int_1^e \sqrt{1+(\tfrac{1}{2}x - \tfrac{1}{2}x^{-1})^2}\, dx = \int_1^e \sqrt{1 + \tfrac{1}{4}x^2 - \tfrac{1}{2} + \tfrac{1}{2}x^{-2}}\, dx$$

$$= \int_1^e \sqrt{\tfrac{1}{4}x^2 + \tfrac{1}{2} + \tfrac{1}{4}x^{-2}}\, dx = \int_1^e \sqrt{(\tfrac{1}{2}x + \tfrac{1}{2}x^{-1})^2}\, dx$$

$$= \int_1^e \frac{1}{2}x + \frac{1}{2}x^{-1}\, dx = \frac{1}{4}x^2 + \frac{1}{2}\ln|x| \Big|_1^e = \frac{1}{4}e^2 + \frac{1}{2} - \frac{1}{4}$$

$$= \boxed{\frac{e^2+1}{4}}$$

#3 $f(x) = x^3 + \frac{1}{12}x^{-1}$ on $[1,3]$ $f'(x) = 3x^2 - \frac{1}{12}x^{-2}$

$$AL = \int_1^3 \sqrt{1+(3x^2 - \tfrac{1}{12}x^{-2})}\, dx = \int_1^3 \sqrt{1 + 9x^4 - \tfrac{1}{2} + \tfrac{1}{144}x^{-4}}\, dx$$

$$= \int_1^3 \sqrt{9x^4 + \tfrac{1}{2} + \tfrac{1}{144}x^{-4}}\, dx = \int_1^3 \sqrt{(3x^2 + \tfrac{1}{12}x^{-2})}\, dx$$

$$= \int_1^3 3x^2 + \frac{1}{12}x^{-2}\, dx = x^3 - \frac{1}{12}x^{-1} \Big|_1^3 = (27 - \tfrac{1}{36}) - (1 - \tfrac{1}{12})$$

$$= \boxed{\frac{469}{18}}$$

#4 a) $\int \cosh x\, dx = \int \frac{1}{2}e^x + \frac{1}{2}e^{-x}\, dx = \frac{1}{2}e^x - \frac{1}{2}e^{-x} + c = \sinh x + c$

b) $\frac{d}{dx}(\cosh x) = \frac{d}{dx}(\frac{1}{2}e^x + \frac{1}{2}e^{-x}) = \frac{1}{2}e^x - \frac{1}{2}e^{-x} = \sinh x$

c) $1 + (\sinh x)^2 = 1 + (\frac{1}{2}e^x - \frac{1}{2}e^{-x})^2 = 1 + \frac{1}{4}e^{2x} - \frac{1}{2} + \frac{1}{4}e^{-2x}$

$= \frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x} = (\frac{1}{2}e^x + \frac{1}{2}e^{-x})^2 = (\cosh x)^2$

d)

$$AL = \int_0^{\ln 2} \sqrt{1+(\sinh x)^2}\, dx = \int_0^{\ln 2} \sqrt{(\cosh x)^2}\, dx = \int_0^2 \cosh x\, dx$$

$$= \sinh x \Big|_0^{\ln 2} = (\tfrac{1}{2}(2) - \tfrac{1}{2} \cdot \tfrac{1}{2}) - (\tfrac{1}{2} - \tfrac{1}{2}) = \boxed{3/4}$$

O. $1+\left(ax^n-\frac{1}{4a}x^{-n}\right)^2 = 1+a^2x^{2n}-\frac{1}{2}+\frac{1}{16a^2}x^{-2n}$

$$= a^2x^{2n}+\frac{1}{2}+\frac{1}{16a^2}x^{-2n}$$

$$= \left(ax^n+\frac{1}{4a}x^{-n}\right)^2$$