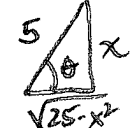


# Day 24 Calculus II

#1  $\int \sqrt{25-x^2} dx$



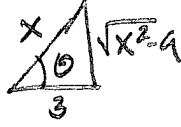
$x = 5 \sin \theta$  ( $x/5 = \sin \theta$ )  
 $dx = 5 \cos \theta d\theta$   
 $\sqrt{25-x^2} = 5 \cos \theta$

$$= \int 5 \cos \theta \cdot 5 \cos \theta d\theta = 25 \int \cos^2 \theta d\theta = 25 \int \frac{1}{2} + \frac{1}{2} \cos(2\theta) d\theta$$

$$= 25 \left[ \frac{\theta}{2} + \frac{1}{4} \cos(2\theta) \right] + C = 25 \left[ \frac{\theta}{2} + \frac{1}{2} \sin \theta \cos \theta \right] + C$$

$$= \frac{25}{2} \left[ \arcsin\left(\frac{x}{5}\right) + \frac{x \sqrt{25-x^2}}{25} \right] + C = \frac{25}{2} \arcsin\left(\frac{x}{5}\right) + \frac{x \sqrt{25-x^2}}{2}$$

#2  $\int \frac{1}{x^2 \sqrt{x^2-9}} dx$

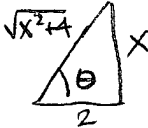


$x = 3 \sec \theta$   
 $dx = 3 \sec \theta \tan \theta d\theta$   
 $\sqrt{x^2-9} = 3 \tan \theta$

$$= \int \frac{3 \sec \theta \tan \theta d\theta}{9 \sec^2 \theta \cdot 3 \tan \theta}$$

$$= \frac{1}{9} \int \frac{1}{\sec \theta} d\theta = \frac{1}{9} \int \cos \theta d\theta = \frac{1}{9} \sin \theta + C = \frac{1}{9} \frac{\sqrt{x^2-9}}{x} + C$$

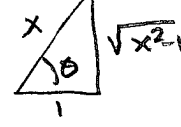
#3  $\int_0^2 \frac{1}{(x^2+4)^{3/2}} dx$



$x = 2 \tan \theta$        $x=0=2 \tan \theta \Rightarrow \tan \theta=0 \Rightarrow \theta=0$   
 $dx = 2 \sec^2 \theta d\theta$        $x=2=2 \tan \theta \Rightarrow \tan \theta=1 \Rightarrow \theta=\pi/4$   
 $\sqrt{x^2+4} = 2 \sec \theta$

$$= \int_0^{\pi/4} \frac{2 \sec^2 \theta d\theta}{(2 \sec \theta)^3} = \frac{1}{4} \int_0^{\pi/4} \frac{1}{\sec \theta} d\theta = \frac{1}{4} \int_0^{\pi/4} \cos \theta d\theta = \frac{1}{4} \sin \theta \Big|_0^{\pi/4} = \frac{1}{4} \cdot \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{2}}{8}$$

#4  $\int \frac{1}{x^3 \sqrt{x^2-1}} dx$



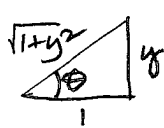
$x = \sec \theta$   
 $dx = \sec \theta \tan \theta d\theta$   
 $\sqrt{x^2-1} = \tan \theta$

$$\int \frac{\sec \theta \tan \theta d\theta}{\sec^3 \theta \tan \theta} = \int \frac{1}{\sec^2 \theta} d\theta = \int \cos^2 \theta d\theta = \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta$$

↙ As usual

$$= \frac{1}{2} \theta + \frac{1}{2} \sin \theta \cos \theta + C = \frac{1}{2} \arctan(\sqrt{x^2-1}) + \frac{1}{2} \frac{\sqrt{x^2-1}}{x} + C$$

#5  $\int \frac{y^4}{1+y^2} dy$



$y = \tan \theta$   
 $dy = \sec^2 \theta d\theta$   
 $\sqrt{1+y^2} = \sec \theta$

$$= \int \frac{\tan^4 \theta \sec^2 \theta d\theta}{\sec^2 \theta} = \int \tan^4 \theta d\theta = \frac{1}{3} \tan^3 \theta - \int \tan^2 \theta d\theta$$

$$= \frac{1}{3} \tan^3 \theta - \int \sec^2 \theta - 1 d\theta = \frac{1}{3} \tan^3 \theta - \tan \theta + \theta + C$$

$$= \frac{1}{3} y^3 - y + \arctan y + C$$

Day 24

#2

#6  $\int \frac{6x-20}{x^2-5x} dx$

$$\frac{6x-20}{x(x-5)} = \frac{A}{x} + \frac{B}{x-5} = \frac{Ax-5A+Bx}{x(x-5)}$$

$$\begin{cases} \text{Coeff: } A+B=6 \\ \text{const: } -5A=-20 \end{cases} \Rightarrow A=4, B=2$$

$$= \int \frac{4}{x} + \frac{2}{x-5} dx = 4 \ln|x| + 2 \ln|x-5| + C$$

#7  $\int \frac{7x-18}{x^2-5x+6} dx$

$$\frac{7x-18}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2} = \frac{Ax-2A+Bx-3B}{(x-3)(x-2)}$$

$$\begin{cases} \text{Coeff: } A+B=7 \rightarrow 3A+3B=21 \\ \text{const: } -2A-3B=-18 \end{cases} \Rightarrow \frac{-2A-3B=-18}{A=3} \text{ so } B=4$$

$$\Rightarrow \int \frac{3}{x-3} + \frac{4}{x-2} dx = 3 \ln|x-3| + 4 \ln|x-2| + C$$