Math 121 Day 27 – Use Correct Notation

"$0\cdot\infty$" Reciprocal process

#1a) $\displaystyle \lim_{x\to0^+} x^2\ln x = \lim_{x\to0^+}\frac{\overset{\to-\infty}{\ln x}}{\underset{\to\infty}{1/x^2}} \overset{L'H}{=} \lim_{x\to0^+}\frac{\frac{1}{x}}{-\frac{2}{x^3}} = \lim_{x\to0^+}\frac{-x^2}{2} = \boxed{0}$

b) $\displaystyle \lim_{x\to\infty} \overset{\to\infty}{x}\,\overset{\to0}{\tan(1/x)} = \lim_{x\to\infty}\frac{\overset{\to0}{\tan(1/x)}}{\underset{\to0}{1/x}} = \lim_{x\to\infty}\frac{-1/x^2\sec^2(1/x)}{-1/x^2}$

Reciprocal process

$\displaystyle = \lim_{x\to\infty}\sec^2(1/x) = \sec 0 = \boxed{1}$

"1^∞" use log process

c) $\displaystyle \lim_{x\to\infty}(1-2/x)^x = y$

$\displaystyle \ln y = \ln\lim_{x\to\infty}(1-2/x)^x \overset{\text{cont}}{=} \lim_{x\to\infty} \overset{\to\infty}{x}\,\ln\overset{\to0}{(1-2/x)} = \lim_{x\to\infty}\frac{\ln(1-2/x)}{1/x}$

$\displaystyle \overset{L'H}{=} \lim_{x\to\infty}\frac{\left(\frac{1}{1-2/x}\right)\left(\frac{2}{x^2}\right)}{-1/x^2} = \lim_{x\to\infty} -\left(\overset{\to1}{\frac{1}{1-2/x}}\right)(2) = -2$

$\displaystyle \ln y = -2 \Rightarrow y = \boxed{e^{-2} = \lim_{x\to\infty}(1-2/x)^x}$ ← Give the final answer

d) $\displaystyle \lim_{x\to0}\frac{\overset{\to0}{\sin kx}}{\underset{\to0}{\arcsin x}} = \lim_{x\to0}\frac{k\cos(kx)}{\frac{1}{\sqrt{1-x^2}}} = \frac{k}{1} = \boxed{k}$

e) $\displaystyle \lim_{x\to0^+} x^{3x} = y \qquad 0^0$

$\displaystyle \ln y = \ln\lim_{x\to0^+}x^{3x} = \lim_{x\to0^+} \overset{\to0}{3x}\,\overset{\to-\infty}{\ln x} = \lim_{x\to0^+}\frac{\overset{\to-\infty}{3\ln x}}{\underset{\to\infty}{1/x}}$

$\displaystyle \overset{L'H}{=} \lim_{x\to0^+}\frac{\frac{3}{x}}{-1/x^2} = \lim_{x\to0^+} -3x = 0$

$\displaystyle \ln y = 0 \Rightarrow y = e^0 = \boxed{1 = \lim_{x\to0^+}x^{3x}}$ ← Give the final answer

f) $\displaystyle \lim_{x\to\infty}\ln(2x-2)-\ln(x+7) = \lim_{x\to\infty}\ln\left(\overset{\to2}{\frac{2x-2}{x+7}}\right) = \ln 2$

#2 $\displaystyle \int_0^\infty\frac{1}{(x+1)^3}\,dx = \lim_{b\to\infty}\int_0^b(x+1)^{-3}\,dx = \lim_{b\to\infty}-\frac{1}{2}(x+1)^{-2}\Big|_0^b$

use limits

$\displaystyle = \lim_{b\to\infty}-\frac{1}{2}\left[\overset{\to0}{\frac{1}{(b+1)^2}}-\frac{1}{1^2}\right] = \frac{1}{2}$

#3 $\int_0^\infty \frac{4}{\sqrt[3]{x+1}}\,dx = \lim_{b\to\infty} \int_0^b 4(x+1)^{-1/3}dx = \lim_{b\to\infty} 4\cdot\frac{3}{2}(x+1)^{2/3}\Big|_0^b$

$= \lim_{b\to\infty} 6\left[(b+1)^{2/3} - 1^{2/3}\right] = \boxed{\infty}$ Diverges

#4 $\int_0^\infty 2xe^{-x^2}dx = \lim_{b\to\infty}\int_0^b 2xe^{-x^2}dx = \lim_{b\to\infty} -e^{-x^2}\Big|_0^b = \lim_{b\to\infty} -e^{-b^2}+e^0$

$u=-x^2$
$du=-2xdx$
$-du=2xdx$

$\int -e^u du = -e^u$

$= 0+1 = \boxed{1}$

5 $\int_0^\infty \frac{4}{1+x^2}dx = \lim_{b\to\infty}\int_0^b \frac{4}{1+x^2}dx = \lim_{b\to\infty} 4\arctan(x)\Big|_0^b$

$= \lim_{b\to\infty} 4\arctan(b) - 4\arctan(0) = 4(\pi/2) - 4(0)$
$= \boxed{2\pi}$

#6 Given $y=(x+2)^2$ $\sqrt{y}=x+2$ so $x=\sqrt{y}-2$ $D = 60\ \text{lbs}/\text{ft}^3$

$A(y) = \pi x^2 = \pi(\sqrt{y}-2)^2$

where the liquid is

$W = D\int_a^b A(y)(l-y)\,dy = 60\int_0^1 \pi(\sqrt{y}-2)^2(7-y)\,dy$ where the liquid goes

#7 Bonus

0^0 ... log process

$\lim_{x\to0^+}(\tan x)^x = y$

cont

$\ln y = \ln \lim_{x\to0^+}(\tan x)^x = \lim_{x\to0^+}\ln(\tan x)^x = \lim_{x\to0^+} x\ln\tan x$

$= \lim_{x\to0^+}\frac{\ln\tan x}{1/x} \overset{L'H}{=} \lim_{x\to0^+}\frac{\frac{1}{\tan x}\cdot\sec^2 x}{-1/x^2} = \lim_{x\to0^+} -\frac{x^2\sec^2 x}{\tan x}$

product Rule $\longrightarrow \overset{L'H}{=} \lim_{x\to0^+}\frac{-2x\sec^2 x - x^2\,2\sec^2 x\tan x}{\sec^2 x} = \lim_{x\to0^+} -2x - 2\tan x = 0$

$\ln y = 0$
$y = e^0 = 1$

So $\lim_{x\to0^+}(\tan x)^x = \boxed{y=1}$