Math 131 Day 29

#1 $\int \frac{8}{x^2+2x-3}\,dx = \int \frac{2}{x+1} - \frac{2}{x-3}\,dx = 2\ln|x+1| - 2\ln|x-3| + C$

$= 2\ln\left|\frac{x+1}{x-3}\right| + C$

a) $\frac{8}{x^2+2x-3} = \frac{A}{x+1} + \frac{B}{x-3} = \frac{Ax-3A+Bx+B}{(x+1)(x-3)}$

x: $A+B=0$

const: $-3A+B=8$

$-4A = -8 \quad A=2,\ B=-2$

b) $\int_2^\infty \frac{8}{x^2+2x-3}\,dx = \lim_{b\to\infty}\int_2^b \frac{8}{x^2+2x-3}\,dx = \lim_{b\to\infty} 2\ln\left|\frac{x-1}{x+3}\right|\Big|_2^b$

$= \lim_{b\to\infty} 2\ln\left|\frac{b-1}{b+3}\right| - 2\ln\frac{1}{5} = \lim_{b\to\infty} 2\ln\left|\frac{1-1/b}{1+3/b}\right| + 2\ln 5$

inside the log $\frac{\infty}{\infty}$

Divide by b

$= 2\ln|1| + 2\ln 5 = 2\ln 5$ or $\ln 25$

c) $\int_0^1 \frac{8}{x^2+2x-3}\,dx = \lim_{b\to 1^-}\int_0^b \frac{8}{x^2+2x-3}\,dx = \lim_{b\to 1^-} 2\ln\left|\frac{x-1}{x+3}\right|\Big|_0^b$

improper at $x=1$ (division by 0)

$\to 0$ ← Remember $\lim_{x\to 0^+} \ln x = -\infty$

$= \lim_{b\to 1^-} 2\ln\left|\frac{b-1}{b+3}\right| - 2\ln\frac{1}{3} = -\infty$

$p>1$

#2a) $\int_1^\infty \frac{1}{x^{5/4}}\,dx = \frac{1}{5/4-1} = 4$

(b) $\int_1^\infty \frac{1}{x^{2/3}}\,dx = +\infty \quad (p = 2/3 \le 1)$

c) $\int_1^\infty \frac{2}{x^7}\,dx = 2\int_1^\infty \frac{1}{x^7}\,dx = 2\left(\frac{1}{7-1}\right) = 2/6 = 1/3$ ($p=7>1$)

d) $\int_1^\infty \frac{1}{x^{-15}}\,dx$ Diverges $(p<1)$

#3) a) $\int \frac{4x^3}{(1+x^4)^2} = \int \frac{1}{u^2}\,du = -\frac{1}{u} + C = \frac{-1}{1+x^4} + C$

$u = 1+x^4,\ du = 4x^3\,dx$

From work below

b) $\int_{-\infty}^\infty \frac{4x^3}{(1+x^4)^2}\,dx = \int_{-\infty}^0 \frac{4x^3}{(1+x^4)^2}\,dx + \int_0^\infty \frac{4x^3}{(1+x^4)}\,dx = -1+1=0$

Do each part separately

$\int_{-\infty}^0 \frac{4x^3}{(1+x^4)^2}\,dx = \lim_{b\to-\infty}\int_b^0 \frac{4x^3}{(1+x^4)^2}\,dx = \lim_{b\to-\infty} \frac{-1}{1+x^4}\Big|_b^0$

$= \lim_{b\to-\infty} \frac{-1}{1} + \frac{1}{1+b^4} = -1$ ($\frac{1}{1+b^4} \to 0$)

Day 29

$$\int_0^\infty \frac{4x^3}{(1+x^4)^2}\,dx = \lim_{b\to\infty}\int_0^b \frac{4x^3}{(1+x^4)^2}\,dx = \lim_{b\to\infty} \frac{-1}{1+x^4}\Big|_0^b = \lim_{b\to\infty} \overbrace{\frac{-1}{1+b^4}}^{0} + \frac{1}{1+0^4} = 1$$

#4 Integrals are improper where the integrand is not defined
You must use limits to evaluate them

a) $\int_3^4 \frac{1}{(x-3)^{3/2}}\,dx = \lim_{b\to 3^+}\int_b^4 \frac{1}{(x-3)^{3/2}}\,dx = \lim_{b\to 3^+} \frac{-2}{(x-3)^{1/2}}\Big|_b^4$ ← evaluate at each pt

problem → 3

$= \lim_{b\to 3^+} \frac{-2}{(4-3)^{1/2}} + \overbrace{\frac{2}{(b-3)^{1/2}}}^{+\infty} = +\infty$ Diverges

denominator → 0

b) $\int_3^4 \frac{1}{(x-3)^{2/3}} = \lim_{b\to 3^+}\int_b^4 \frac{1}{(x-3)^{2/3}}\,dx = \lim_{b\to 3^+} 3(x-3)^{1/3}\Big|_b^4$

same issue

$= \lim_{b\to 3^+} 3(4-1)^{1/3} - \overbrace{3(b-3)^{1/3}}^{\to 0} = 3 - 0 = 3$

c) $\int_0^{\pi/2} \sec x\tan x\,dx = \lim_{b\to\frac{\pi}{2}^-}\int_0^b \sec x\tan x\,dx = \lim_{b\to\frac{\pi}{2}^-} \overbrace{\sec(b)}^{+\infty} - \overbrace{\sec(0)}^{=1}$

not defined at $\frac{\pi}{2}$ — improper

$= \infty$ Diverges

#5 Again, integrals are improper where there is division by 0 (Vertical Asymptote)

a) $\int_1^3 \frac{x-3}{x^2+3x-4}\,dx = \int_1^3 \frac{x-3}{(x-1)(x+4)}\,dx = \lim_{b\to 1^+}\int_b^3 \frac{x-3}{(x-1)(x+4)}\,dx$

Improper ← ok not in interval !

b) $\int_1^3 \frac{x-3}{x^2+4}\,dx$ ← "proper"

Never 0

c) $\int_1^3 \frac{-3}{x-2}\,dx = \lim_{a\to 2^-}\int_1^a \frac{-3}{x-2}\,dx + \lim_{a\to 2^+}\int_a^2 \frac{-3}{x-2}\,dx$

$x=2$ is in the interval from 1 to 3