Math 131 Day 30

#1 a) $a_n = 2+(-1)^n$: $a_1 = 2+(-1)^1 = 1$. Similarly $a_2 = 3$, $a_3 = 1$, $a_4 = 3$

p604 #12, 14, 22

b) $a_n = n + 1/n$: $a_1 = 1 + 1/1 = 2$, $a_2 = 1\frac{1}{2}$, $a_3 = 1\frac{1}{3}$, $a_4 = 1\frac{1}{4}$

c) $a_{n+1} = a_n + a_{n-1}$ $a_0 = 1$, $a_1 = 1$, $a_2 = a_0 + a_1 = 2$, $a_3 = 3$, $a_4 = 5$, $a_6 = 8$, .

d) $\{1/4, 2/9, 3/16, 4/25, \dots\}$ $a_n = n/(n+1)^2$

e) $\{64, 32, 16, 8, 4, \dots\}$ $a_{n+1} = \frac{1}{2} a_n$, $a_1 = 64$

#2 a) $\frac{10!}{8!} = \frac{1\cdot 2 \cdots 8 \cdot 9 \cdot 10}{\cancel{1 \cdot 2 \cdots 8}} = 9 \cdot 10 = 90$

b) $\frac{5!}{7!} = \frac{5!}{5! \cdot 6 \cdot 7} = \frac{1}{6 \cdot 7} = \frac{1}{42}$

c) $\frac{(n+2)!}{n!} = \frac{\cancel{n!}(n+1)(n+2)}{\cancel{n!}} = (n+1)(n+2)$ or n^2+3n+2

d) $\frac{3^2 \cdot (n-1)!}{(n+1)!} = \frac{3^2 \cdot \cancel{(n-1)!}}{\cancel{(n-1)!}\,(n)(n+1)} = \frac{9}{n^2+n}$

e) $\frac{4 \cdot \cancel{4} \cdot 4 \cdot 4^2}{\cancel{4} \cdot 3 \cdot 2 \cdot 1} = \frac{32}{3}$

#3 a) $\lim\limits_{n\to\infty} (1 + 2/n)^n = e^2$ ← key limit

"0^0"

b) $\lim\limits_{n\to\infty} (1/n)^{1/n} = \lim\limits_{x\to\infty} (1/x)^{1/x} = y = \boxed{1}$

$\ln y = \ln \lim\limits_{x\to\infty} (1/x)^{1/x} \overset{\text{cont}}{=} \lim\limits_{x\to\infty} \ln (1/x)^{1/x} = \lim\limits_{x\to\infty} \frac{\ln(1/x)^{\to -\infty}}{x_{\to\infty}}$

$\overset{L'H}{=} \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$, so $y = e^0 = 1$

c) $\lim\limits_{n\to\infty} \overset{\to\infty\cdot 0}{n \sin(1/n)} = \lim\limits_{x\to\infty} \frac{\overset{\to 0}{\sin(1/x)}}{\underset{\to 0}{1/x}} \overset{L'H}{=} \lim\limits_{x\to\infty} \frac{\cos(1/x)\cdot(-1/x^2)}{(-1/x^2)} = \boxed{1}$

d) $\lim\limits_{n\to\infty} (e/\pi)^n = 0$ ← key limit $|r| = |e/\pi| < 1$

#4 $\lim\limits_{n\to\infty} \ln n + \ln(\sin 1/n) = \lim\limits_{n\to\infty} \ln(n \sin 1/n) \overset{\text{cont}, x}{=} \ln \lim\limits_{x\to\infty} \boxed{x \sin(1/x)}$ use (3c)

$= \ln 1 = 0$

#5 a) Bounded (B = 3), eventually monotonic at N = 2

b) non-decreasing, monotonic, Bounded (B = 3, 2?)

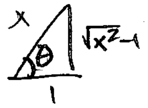
c) Bounded B = 1/2, eventually monotonic at N = 3

d) Bounded B = 1

#6 $\int_1^2 \frac{1}{\sqrt{x^2-1}}\,dx = \lim_{a\to 1^+} \int_a^2 \frac{1}{\sqrt{x^2-1}}\,dx = \lim_{a\to 1^+} \ln\left|x+\sqrt{x^2-1}\right|\Big|_a^2$

$= \lim_{a\to 1^+} \ln\left|1+\sqrt{3}\right| - \ln\left|\overset{\nearrow 1}{a} + \overset{\nearrow 0}{\sqrt{a^2-1}}\right|$

$= \ln\left|1+\sqrt{3}\right|$



$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = \tan\theta$

$\int \frac{\sec\theta\tan\theta}{\tan\theta}\,d\theta = \int \sec\theta\,d\theta$

$= \ln\left|\sec\theta+\tan\theta\right| + C$

$= \ln\left|x+\sqrt{x^2-1}\right| + C$