Day 31 Math 131

#1a $\lim_{n\to\infty} (1-6/n)^{2n/3} = \lim_{n\to\infty}\left[\left(\underbrace{(1-6/n)^n}_{\to e^{-6}}\right)\right]^{2/3} = (e^{-6})^{2/3} = e^{-4}$

b) $\lim_{n\to\infty} n^{8/n} = \lim_{n\to\infty} (\underbrace{n^{1/n}}_{\to 1})^8 = 1^8 = 1$

c) $\lim_{n\to\infty} 2^{3n}\cdot 9^{-n} = \lim_{n\to\infty} \frac{(2^3)^n}{9^n} = \lim_{n\to\infty}\left(\frac{8}{9}\right)^n = 0$; $|8/9|<1$

d) $\lim_{n\to\infty} (-2)^{-n} = \lim_{n\to\infty}\left(\frac{1}{-2}\right)^n = 0$; $|-1/2|<1$

#2 $\frac{1}{x^2+5x+6} = \frac{A}{x+2} + \frac{B}{x+3} = \frac{Ax+3A+Bx+2B}{x^2+5x+6}$ Improper at $x=-2,-3$

x: $A+B=0 \Rightarrow A=-B$

Const: $3A+2B=1 \Rightarrow -3B+2B=-B=1$, $B=-1$, $A=1$.

$\int_{-2}^{0} \frac{1}{x^2+5x+6}\,dx = \lim_{b\to -2^+} \int_b^0 \frac{1}{x+2} - \frac{1}{x+3}\,dx = \lim_{b\to -2^+} \ln|x+2| - \ln|x+3| \Big|_b^0$

$= \lim_{b\to -2^+} \ln 2 - \ln 3 - (\ln(\underbrace{b+2}_{\to 0^+}) - \ln(\underbrace{b+3}_{\to 1})) = +\infty$ Diverges

$\lim_{x\to 0^+} \ln x = -\infty$

#3 $\lim_{n\to\infty} (n+2)^{1/n} = y = 1$ "∞^0"

$\ln y = \ln \lim_{n\to\infty} (n+2)^{1/n} = \lim_{x\to\infty} \frac{\ln(x+2)}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x+2}}{1} = 0 \Rightarrow y = e^0 = 1$

b) $\lim_{n\to\infty} n^2 \sin(1/n) = \lim_{x\to\infty} \frac{\sin(1/x)}{1/x^2} = \lim_{x\to\infty} \frac{-1/x^2\cos(1/x)}{-2/x^3} = \lim_{x\to\infty} \frac{x\cos(1/x)}{2}$ "$\infty\cdot 0$"

$= \infty$ Diverges

#4 a) $a_n = 2-3/n$; $f(x) = 2-3/x$; $f'(x) = 3/x^2 > 0$ So the sequence is increasing

b) $a_n = n\ln n$; $f(x) = x\ln x$; $f'(x) = \ln x + x\cdot 1/x$

$= \ln x + 1 > 0$ for $x \geq 1$

So the sequence is increasing