Math 131 Day 3#3

#1 a) $\lim_{n\to\infty} \frac{3n+1}{2n+5} = \lim_{n\to\infty} \frac{3+1/n}{2+5/n} = \frac{3}{2} \neq 0$. So by the nth term test for divergence, the series $\sum_{n=1}^{\infty} \frac{3n+1}{2n+5}$ diverges

b) $\lim_{n\to\infty} \frac{n}{2n^2+1} = \lim_{n\to\infty} \frac{1/n}{2+1/n^2} = 0$. So the nth term test does not apply

c) $\lim_{n\to\infty} (1.1)^n = \infty \neq 0$ (key limit $|r|>1$). So by the nth term test $\sum_{n=1}^{\infty} (1.1)^n$ diverges

d) $\lim_{n\to\infty} \left(1+\frac{4}{n}\right)^n = e^4 \neq 0$ (key limit). So by the nth term test, $\sum_{n=1}^{\infty} (1+4/n)^n$ diverges

e) $\lim_{n\to\infty} \sqrt[n]{n} = 1 \neq 0$. By the nth term test, $\sum_{n=1}^{\infty} \sqrt[n]{n}$ diverge

f) $\lim_{n\to\infty} \frac{n!}{n^n} = 0$. The nth term test for divergence does not apply.

#2 a) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges by the p-series test since $p = 1/2 \leq 1$

Rewrite

b) $\sum_{n=1}^{\infty} \frac{1}{e^{2n}} = \sum_{n=1}^{\infty} \left(\frac{1}{e^2}\right)^n$ converges by the geometric series test since $|r| = \frac{1}{e^2} < 1$

c) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by the p-series test since $p = 3 > 1$

(*d*) Use the integral test. The corresponding function is $f(x) = \frac{1}{9+x^2}$ and is clearly positive and continuous on $[1,\infty)$. Notice that $f'(x) = \frac{-2x}{(9+x^2)^2} < 0$ for all x in $[1,\infty)$. So f is decreasing. The improper integral (is an inverse tangent integral):

$$\int_1^{\infty} \frac{1}{9+x^2}\,dx = \lim_{b\to\infty} \frac{1}{3}\arctan\frac{x}{3}\Big|_1^b = \lim_{b\to\infty} \frac{1}{3}\arctan\frac{b}{3} - \frac{1}{3}\arctan\frac{1}{3} = \frac{1}{3}\left[\frac{\pi}{2} - \arctan\frac{1}{3}\right].$$

Since the integral converges, by the integral test the series $\sum_{n=1}^{\infty} \frac{1}{9+n^2}$ also converges.

#2e.) $\sum_{n=1}^{\infty} \frac{3}{n^2+7n+10}$ ... Integral Test: $f(x) = \frac{3}{x^2+7x+10}$ is cont, pos for $x \geq 1$ and decreasing ... the numerator is constant and $x^2+7x+10$ increases as x increases, so the whole function decreases

$$\int_1^{\infty} \frac{3}{x^2+7x+10}dx = \lim_{b\to\infty} \int_1^b \frac{1}{x+2} - \frac{1}{x+5}\,dx = \lim_{b\to\infty} \ln\left|\frac{x+2}{x+5}\right| \Big|_1^b$$

$$= \lim_{b\to\infty} \ln\left|\frac{b+2}{b+5}\right| - \ln 3/6 = \ln 1 + \ln 2 = \ln 2$$

Since the integral converges so does $\sum \frac{3}{n^2+7n+10}$ by Integral Test

2f) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^3}} = \sum \frac{3}{n^{3/2}}$ Converges by p-series test $(p = 3/2 > 1)$

2g) $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ ... $f(x) = \frac{x}{2x^2+1}$; $f'(x) = \frac{2x^2+1-x(4x)}{(2x^2+1)^2} = \frac{1-2x^2}{(2x^2+1)^2} < 0$ for $x \geq 1$

So f is positive, cont, decreasing ... apply integral test

$$\int_1^{\infty} \frac{x}{2x^2+1} = \lim_{b\to\infty} \int_1^b \frac{x}{2x^2+1}dx = \lim_{b\to\infty} \frac{1}{4}\ln|2x^2+1| \Big|_1^b$$

$$= \lim_{b\to\infty} \frac{1}{4}\underbrace{\ln(2b^2+1)}_{\to\infty} - \ln 3 = \infty \text{ Diverges}$$

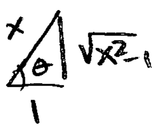
So by the Integral Test $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ also diverges

2h) $\sum_{n=1}^{\infty} \frac{1}{n^{1.00001}}$ converges by the p-series test $(p = 1.00001 > 1)$

2i) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$ ... Integral test $f(x) = \frac{1}{\sqrt{x^2-1}}$ is pos & cont for $x \geq 2$

$f'(x) = -1/2(x^2-1)^{-3/2}(2x) < 0$ for $x \geq 2$ (Decr)

$$\int_2^{\infty} \frac{1}{\sqrt{x^2-1}}dx = \lim_{b\to\infty} \ln|x+\sqrt{x^2-1}| \Big|_2^b = \lim_{b\to\infty} \underbrace{\ln|b+\sqrt{b^2-1}|}_{\to\infty} - \ln|2+\sqrt{3}|$$



$x = \sec\theta$
$dx = \sec\theta\tan\theta\,d\theta$
$\sqrt{x^2-1} = \tan\theta$

$$\int \frac{1}{\sqrt{x^2-1}}dx = \int \frac{\sec\theta\cancel{\tan\theta}\,d\theta}{\cancel{\tan\theta}} = \ln|\sec\theta+\tan\theta| + c$$
$$= \ln|x+\sqrt{x^2-1}| + c$$

$= \infty$ Diverges

Since $\int_2^{\infty} \frac{1}{\sqrt{x^2-1}}dx$ diverges so does $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$ by the integral test.