Day 34 Math 131

#1 $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k+10}}$. Use the integral test. $f(x) = \frac{1}{(x+10)^{1/3}}$ on $[1,\infty)$

p638 #20

$f(x)$ is positive and continuous. As x increases the denominator gets larger and numerator stays the same. So $f(x)$ decreases.... integral test applies

or show $f'(x)<0$

$$\int_1^{\infty} \frac{1}{\sqrt[3]{x+10}}\,dx = \lim_{b\to\infty} \int_1^b (x+10)^{-1/3}\,dx = \lim_{b\to\infty} \frac{3}{2}(x+10)^{2/3}\Big|_1^b$$

$$= \lim_{b\to\infty} \frac{3}{2}\left[\underbrace{(b+10)^{2/3}}_{\to\infty} - (11)^{2/3}\right] = \infty \text{ Diverges}$$

Since the integral diverges, by the integral test the series $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k+10}}$ also diverges

#2

p638 #24 $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$: Integral test $f(x) = \frac{1}{x(\ln x)^2}$ on $[2,\infty)$

$f(x)$ is positive and continuous and as x increases, the denominator increases but the numerator stays the same. So f is decreasing. Apply integral test.

$$\int \frac{1}{x(\ln x)^2}\,dx = \int u^{-2}\,du = -u^{-1} = (\ln x)^{-1} = \frac{-1}{\ln x}$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\text{So } \int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx = \lim_{b\to\infty} \frac{-1}{\ln x}\Big|_2^b = \lim_{b\to\infty} \frac{-1}{\underbrace{\ln(b)}_{\to\infty}} + \frac{1}{\ln 2} = 0 + \frac{1}{\ln 2}$$

The integral converges, so does $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ by the integral test

#3 $\sum_{k=1}^{\infty} \frac{2^k}{k!}$. The terms

p647 #10

$\sum_{k=1}^{\infty} \frac{2^k}{k!}$ positive terms – Apply ratio test

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{2^{k+1}}{(k+1)!}\cdot\frac{k!}{2^k} = \lim_{k\to\infty} \frac{2}{k+1} = 0 < 1$$

By the ratio test, the series converges $(r<1)$

p647 # 14 $\sum_{k=1}^{\infty} \frac{k^k}{k!}$. Terms are positive – powers, factorial apply Ratio test

$$r = \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{(k+1)^{k+1}}{(k+1)!} \cdot \frac{k!}{k^k} = \lim_{k\to\infty} \frac{(k+1)^{\cancel{k+1}}}{\cancel{k+1}} \cdot \frac{1}{k^k}$$

$$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^k = \lim_{k\to 1} (1+1/k)^k = e > 1$$

Since $r = e > 1$, the series diverges by the ratio Test

Review 623 # 26 $\sum_{m=2}^{\infty} \frac{5}{2^m} = 5\cdot\left(\frac{1}{2}\right)^2 + 5\cdot\left(\frac{1}{2}\right)^3 + 5\left(\frac{1}{2}\right)^4 + \cdots$

a $\quad r = 1/2 \quad |r| < 1$

So $\sum_{m=2}^{\infty} 5/2^m = \frac{a}{1-r} = \frac{5\cdot 1/4}{1-1/2} = \frac{5/4}{1/2} = \boxed{5/2}$ (converges)

p 638 # 16 $\sum_{k=1}^{\infty} \frac{\sqrt{k^2+1}}{k}$. Apply Divergence Test

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{\sqrt{k^2+1}}{k} = \lim_{k\to\infty} \frac{\sqrt{1+1/k^2}}{1} = 1 \neq 0$$

$\div$ by $k = \sqrt{k^2}$

By the Divergence Test, the series $\sum_{k=1}^{\infty} \frac{\sqrt{k^2+1}}{k}$ diverges

#7 From the list: The ratio test applies to:

(c), (l), (m), (o), maybe (k)