My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

Practice

Remember Lab 2 tomorrow. After lab review the answers online.

1. Review Section 5.2 which finishes our introduction to the Definite Integral giving a number of its basic properties. Begin to read Section 5.3 on the **Fundamental Theorem of Calculus**. Any theorem with this name must be important.

(*a*) Understand the definition of a **Riemann Sum** (p. 351).

(*b*) Memorize the definition of a **Definite Integral** (p. 351).

(*c*) Understand Theorem 5.2 (p. 352).

(*d*) Remember, geometrically, a definite integral $\int_a^b f(x)\,dx$ is the net area between f and the x-axis on the interval $[a, b]$. We have 'solved' the area problem.

(*e*) Review the properties of Definite Integrals on pages 354-56. These are summarized on page 356.

2. (*a*) Try page 358–359 #5, 7, 21, 23, 25–31 odd.

(*b*) These next problems are easy but use important concepts: Page 359–360 #33, 35, 41, 43, 45.

(*c*) Here are some additional Riemann Sums to practice: Page 360 #49 and 51.

Math 131 Hand In Day 5

0. Do WeBWorK set Day05. (Due Saturday, but some problems may help you with the homework below.)

1. Use an appropriate Riemann sum to evaluate $\int_2^3 (x^2 - 4)\,dx$. Show all your work. (Compare to Lab 2, problem 3).

2. Let $f(x) = x^2 + x$ on $[0, 2]$.

(*a*) Explain why Upper(n) = Right(n) and Lower(n) = Left(n).

(*b*) Determine Upper(n). Show the details.

(*c*) Determine $\int_0^2 (x^2 + x)\,dx$.

(*d*) **Extra Credit:** Show that Lower(n) = Upper$(n) - \frac{16}{n}$. Comment: Written this way makes it easy to see why $\lim_{n\to\infty}$ Upper(n) = $\lim_{n\to\infty}$ Lower(n).

3. Use geometry (not Riemann sums) to determine the following definite integrals. Hint: See Example 3 in Section 5.2. Sketch a graph of the function to illustrate the region.

(*a*) $\displaystyle\int_0^2 (6 - 3x)\,dx$ (*b*) $\displaystyle\int_{-2}^1 -|x|\,dx$ (*c*) $\displaystyle\int_{-3}^3 \sqrt{9 - x^2}\,dx$ (*d*) $\displaystyle\int_0^4 f(x)\,dx$, where $f(x) = \begin{cases} 3, & \text{if } x \le 2 \\ x, & \text{if } x > 2. \end{cases}$

4. Using properties of the integral:

(*a*) Page 359 #38 and 40. Show how you obtained your answer.

(*b*) Page 359 #42(a,c). Use properties of the integral. Show your 'work.'

(*c*) Page 360 #46. Note: There is a typo in the text. Use $I = \displaystyle\int_0^{\pi/2} (2\sin\theta - \cos\theta)\,d\theta = -1$. Use properties of the integral. Note the order of the limits in the problems.

5. Compare to Lab 2, Problem 5. Review it and the answers on line if needed. Make the 'adjustments' necessary (see the lab) to determine the following antiderivatives. (Check your answers by taking the derivative.)

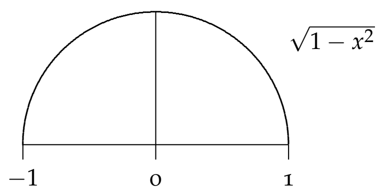
(*a*) $\displaystyle\int \sqrt{7x}\,dx$ (*b*) $\displaystyle\int \sin\frac{\pi x}{3}\,dx$

6. (*a*) What is the area of a semi-circle of radius 1? Give your answer in terms of π and also as a decimal rounded correctly to four places.

(*b*) The rest of this problem uses the online calculator at `https://www.desmos.com/calculator/tgyr42ezjq`. The equation of the semi-circle of radius 1 centered at the origin is $f(x) = \sqrt{1-x^2}$ on the interval $[-1, 1]$ (see figure). We should be able to find the area of this region using calculus. According to our theory, since $f(x) = \sqrt{1-x^2}$ is continuous, it is integrable so

$$\text{Area} = \int_{-1}^{1} \sqrt{1-x^2}\,dx = \lim_{n\to\infty} \text{Right}(n) = \lim_{n\to\infty} \text{Left}(n).$$

So we should be able to approximate the answer using left and right Riemann sums with increasingly large values of n. Follow the instructions to use the calculator at the website above (there's a link at our webpage) to find: Left(5), Right(5), and Midpoint(5). You will have to use the "choice of method" directions at the website. Then determine Left(50), Right(50), and Midpoint(50). Finally Left(100), Right(100), and Midpoint(100). Correctly round to four decimal places. Note: $\sqrt{1-x^2}$ is typed as `sqrt(1-x^2)`. The image below shows the values for 10 subintervals.



n	Left(n)	Right(n)	Midpoint(n)
5			
50			
100			

(*c*) Are these estimates getting closer to your answer in part (a) as n gets larger?