We will finish the discussion of partial fractions by looking at complications that can arise. We will only consider partial fraction problems with linear factors. Read and review Section 7.5 through Examples 1, 2, and 3 (top of page 544). See the online notes that have several examples.

1. (a) **Review** L'Hôpital's Rule: Section 4.7.
   
   (b) **Read** in Section 7.8 about Improper Integrals, concentrating the material on unbounded intervals (570–575).
   
   (c) Try page 549 #13, 15, 19, 27, 29, and 31.

2. You need to keep practicing. Try integrating these rational functions (answers below). These are BETTER than those in the text. **Answers on the back.** Some have three factors. Others have repeated factors.

   \[ \int \frac{4t^2 - 3t - 4}{t^3 - t^2 - 2t} \, dt \quad \int \frac{t + 7}{(t + 1)(t^2 - 4t + 3)} \, dt \quad \int \frac{x + 6}{x^2 - x - 6} \, dx \]

   \[ \int \frac{x}{(x-1)(x+1)(x+2)} \, dx \quad \int \frac{2x^2}{(x-1)^2(x+1)} \, dx \quad \int \frac{4x + 4}{(x-2)^2} \, dx \]

3. Try these similar looking problems.

   \[ \int \frac{10}{25 + x^2} \, dx \quad \int \frac{10x}{25 + x^2} \, dx \quad \int \frac{10}{25 - x^2} \, dx \]


**Hand in next time**

Finish **WeBWorK** Day25 and begin Day26.

1. Try these; (b) and (c) are **WeBWorK** Day 26 problems. (Show your work.)

   \[ \int \frac{2}{x^3 - x} \, dx \quad \int \frac{2x^2}{(x - 1)(x^2 + 1)} \, dx \quad \int -\frac{4x + 4}{(x - 2)^2} \, dx \]

2. Review of l’Hôpital’s rule. In each, make sure to check whether l’Hôpital’s rule applies. Remember to use the following limits when necessary: \( \lim_{x \to -\infty} e^x = 0, \lim_{x \to +\infty} e^x = +\infty, \lim_{x \to +\infty} \ln x = +\infty, \) and \( \lim_{x \to -\infty} \ln x = -\infty. \) Most of these are (similar to) **WeBWorK** Day 26 problems. Do at the same time.

   \( \lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1} \quad \lim_{x \to 1} \frac{\ln(x^2)}{x^2 - 1} \quad \lim_{x \to 0} \frac{\sin(ax)}{\sin(bx)}, \, b \neq 0 \)

   \( \lim_{x \to 0} \frac{x}{\arctan(2x)} \quad \lim_{x \to 0} \frac{(\ln x)^2}{x} \quad \lim_{x \to 0} \frac{\sin^2(3x)}{x^2} \)

   \( \lim_{x \to 1} \frac{x^n - 1}{x - 1}, \, n \geq 2 \) a positive integer \( \lim_{x \to 0} \frac{e^x - 1}{\sin(14x)} \)

3. Test review: \( \int \frac{x^2}{\sqrt{9 - x^2}} \, dx \). You **must** draw the triangle; use it to convert to \( x \).
Answers to Practice Problems

1. (a) \( \int \frac{2}{t} + \frac{1}{t-2} + \frac{1}{t+1} \, dt = 2 \ln |t| + \ln |t-2| + \ln |t+1| + c \)
   
   (b) \( \int \frac{3/4 + 5/4}{t-1} - \frac{2/3}{t-1} \, dt = \frac{3}{4} \ln |t+1| + \frac{5}{4} \ln |t-3| - 2 \ln |t-1| + c \)
   
   (c) \( \int \frac{9/5}{x^3} - \frac{4/5}{x^2} \, dt = x + \frac{9}{5} \ln |x-3| - \frac{4}{5} \ln |x+2| + c \)
   
   (d) \( \int \frac{1/6 + 1/2}{x+1} - \frac{2/3}{x+2} \, dt = \frac{1}{6} \ln |x-1| + \frac{1}{2} \ln |x+1| - \frac{2}{3} \ln |x+2| + c \)
   
   (e) \( \int \frac{3/2}{x-1} + \frac{1}{(x-1)^2} + \frac{1/2}{x+1} \, dt = \frac{3}{2} \ln |x-1| - (x-1)^{-1} + \frac{1}{2} \ln |x+1| + c \)
   
   (f) \( \int -\frac{1}{x-2} - \frac{2}{(x-2)^2} + \frac{1}{x} \, dt = -\ln |x-2| + 2(x-2)^{-1} + \ln |x| + c \)

2. (a) \( 2 \arctan \left( \frac{x}{5} \right) + c \)  
   (b) \( 5 \ln |25 + x^2| + c \)  
   (c) \( \ln \left| \frac{x+5}{x-5} \right| + c \)