My Office Hours: M & W 2:30–4:00, Tu 2:00–3:30, & F 1:30–2:30 or by appointment. **Math Intern:** Sun: 2:00–5:00, 7:00–10pm; Mon thru Thu: 3:00–5:30 and 7:00–10:30pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131F15/index.html`.

Test Monday at 7:40 am

Practice materials online. Review Labs. Extra TA's Sunday with the Math Intern.

☕ *Practice.* Review all of Section 8.5 on the ratio, root, and comparison tests. Begin reading Section 8.6 on Alternating Series.

Hand In Monday after Thanksgiving

Justify your answers with an argument. When using comparison, be sure you explain why the series you are comparing to converges or diverges.

1. Root Test: Page 648 #20 and #22.

2. Determine whether these series converge using comparisons:

(*a*) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+8k+12}$ (*b*) $\displaystyle\sum_{k=1}^{\infty} \frac{3}{4k+\sqrt{k}}$

(*c*) Before we knew the limit comparison test, how would you have done part (a)? Knowing more stuff makes life easier

Eight Tests

1. Ratio Test. Assume that $\displaystyle\sum_{n=1}^{\infty} a_n$ is a series with **positive** terms and let $r = \displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

1. If $r < 1$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.
2. If $r > 1$ or $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > \infty$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.
3. If $r = 1$, then the test is inconclusive. The series may converge or diverge.

2. Root Test. Assume that $\displaystyle\sum_{n=1}^{\infty} a_n$ is a series with **positive** terms and let $r = \displaystyle\lim_{n\to\infty} \sqrt[n]{a_n}$.

1. If $r < 1$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.
2. If $r > 1$ or $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > \infty$, then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.
3. If $r = 1$, then the test is inconclusive. The series may converge or diverge.

3. Limit Comparison Test. Assume that $a_n > 0$ and $b_n > 0$ for all n (or at least all $n \geq k$) and that $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

(1) If $0 < L < \infty$ (i.e., L is a positive, *finite* number), then either $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ *both* converge or *both* diverge.

(2) If $L = 0$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

(3) If $L = \infty$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

4. **Direct Comparison Test.** Assume $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms.
   (*a*) If $0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. (If the bigger series converges, so does the smaller series.)
   (*b*) If $0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges. (If the smaller series diverges, so does the bigger series.)

5. **The Geometric Series Test.**
   (*a*) If $|r| < 1$, then the geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n$ converges to $\dfrac{a}{1-r}$.
   (*b*) If $|r| \geq 1$, then the geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n$ diverges.

6. **The nth term test for Divergence.** If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. (If $\lim_{n\to\infty} a_n = 0$, this test is useless.)

7. **The Integral Test.** If $f(x)$ is a **positive**, **continuous**, and **decreasing** for $x \geq 1$ and $f(n) = a_n$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

8. **The p-series Test.** The p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1. \end{cases}$

99. ANSWERS to Lab 12 Review:
   (*a*) $\displaystyle\sum_{n=1}^{\infty} \frac{5 \cdot n!}{2^n}$. **ARGUMENT:** Factorial: Ratio test. The terms are positive. $r = \displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{5\cdot(n+1)!}{2^{n+1}} \cdot \frac{2^n}{5\cdot n!} = \lim_{n\to\infty} \frac{n+1}{2} = \infty > 1$. Since $r > 1$ by the ratio test the series diverges.
   (*b*) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{1+4n^2}$. **ARGUMENT:** Integral test: Note that $f(x) = \frac{2}{1+4x^2}$ is certainly positive and continuous; it is also decreasing because as x increases, the denominator increases, but the numerator stays the same making the function values smaller. Or use the derivative: $f'(x) = \frac{-16x}{(1+4x^2)^2} < 0$ on $[1,\infty)$. Aside $u^2 = 4x^2$, so $u = 2x$, $du = 2\,dx$. Then $\int \frac{2}{1+4x^2}, dx = \int \frac{1}{1+u^2}\,du = \arctan u = \arctan(2x)$. So

$$\lim_{b\to\infty} \int_1^b \frac{1}{1+4x^2}\,dx = \lim_{b\to\infty} \arctan 2x\Big|_1^b = \lim_{b\to\infty} [\arctan 2b - \arctan 2] = \frac{\pi}{2} - \arctan 2.$$

   So the integral converges. So $\sum \frac{2}{1+4n^2}$ also converges by the integral test.
   (*c*) $\displaystyle\sum_{n=1}^{\infty} 2\arctan(n)$. **ARGUMENT:** The Divergence (nth) term test. $\lim_{n\to\infty} 2\arctan(n) = 2(\pi/2) = \pi \neq 0$. So the series diverges by the nth term test.
   (*d*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^6}}$. **ARGUMENT:** p-series test: Since $p = \frac{6}{5} > 1$, the series converges by by the p-series test.
   (*e*) $4 - \dfrac{8}{9} + \dfrac{16}{81} - \dfrac{32}{729} + \cdots$. **ARGUMENT:** Geometric $a = 4$ and we can get r by dividing the next term by the one before it: $r = \frac{a_{n+1}}{a_n} = \frac{-\frac{8}{9}}{4} = \frac{-2}{9}$. So the sum is $\frac{4}{1+\frac{2}{9}} = \frac{36}{11}$.