Math 131 Day 2

#1) p317 # 32 a) $1+3+5+7+\cdots+99 = \sum_{i=0}^{49} 2i+1 = \sum_{k=1}^{50} 2k-1$

(2) d) $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\cdots+\frac{1}{49\cdot50} = \sum_{k=1}^{49} \frac{1}{k(k+1)}$

(2) # 2) p317 # 34 b) $\sum_{k=1}^{45}(5k-1) = 5\sum_{k=1}^{45} k - \sum_{k=1}^{45} 1 = \frac{5(45)(46)}{2} - 45(1) = 5{,}130$

(2) d) $\sum_{n=1}^{50}(1+n^2) = \sum_{n=1}^{50} 1 + \sum_{n=1}^{50} n^2 = 50(1) + \frac{50(51)(101)}{6} = 42{,}975$

(2) g) $\sum_{p=1}^{35}(2p+p^2) = 2\sum_{p=1}^{35} p + \sum_{p=1}^{35} p^2 = \frac{2(35)(36)}{2} + \frac{35(36)(71)}{6} = 16{,}170$

(2) # 3) $\sum_{i=1}^{n}\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \frac{4}{n^2}\sum_{i=1}^{n} i = \frac{4}{n^2}\frac{(n)(n+1)}{2} = \frac{2n+2}{n} = 2+\frac{2}{n}$

(2) b) $\sum_{i=1}^{n}\frac{i^2-10}{n^3} = \frac{1}{n^3}\sum_{i=1}^{n} i^2 - \frac{1}{n^3}\sum_{i=1}^{n} 10 = \frac{1}{n^3}\frac{(n(n+1)(2n+1))}{6} - \frac{1}{n^3}(n)(10)$

$= \frac{2n^2+3n+1}{6n^2} - \frac{10}{n^2} = \frac{1}{3}+\frac{1}{2n}+\frac{1}{6n^2}-\frac{10}{n^2}$

c) $\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)^2\left(\frac{1}{n}\right) = \sum_{i=1}^{n}\left(1+\frac{2i}{n}+\frac{i^2}{n^2}\right)\left(\frac{1}{n}\right) = \frac{1}{n}\sum_{i=1}^{n} 1 + \frac{2}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2$

(3) $= \frac{1}{n}(n)(1) + \frac{2}{n^2}\frac{(n(n+1))}{2} + \frac{1}{n^3}\frac{(n)(n+1)(2n+1)}{6}$

$= 1 + \frac{n+1}{n} + \frac{(n+1)(2n+1)}{6n^2} = 1+\left(1+\frac{1}{n}\right)+\left(\frac{1}{3}+\frac{1}{2n}+\frac{1}{6n^2}\right)$

$= 2\frac{1}{3} + \frac{1}{n} + \frac{1}{2n} + \frac{1}{6n^2}$

#4) a) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty} 2+\frac{2}{n} = 2$

(3) b) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^2-10}{n^3} = \lim_{n\to\infty}\frac{1}{3}+\frac{1}{2n}+\frac{1}{6n^2}-\frac{10}{n^2} = \frac{1}{3}$

c) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)^2\left(\frac{1}{n}\right) = \lim_{n\to\infty} 2\tfrac{1}{3} + \tfrac{1}{n} + \frac{1}{2n} + \frac{1}{6n^2} = 2\tfrac{1}{3} = \tfrac{7}{3}$

#5

$3(\tan x)^4$

a) $\frac{d}{dx}(3\tan^4 x) = 12\tan^3 x\cdot\sec^2 x$

(4) b) $\frac{d}{dx}(6+\ln(8x^5)) = \frac{1}{8x^5}\cdot 40x^4 = \frac{5}{x}$

6. First $P(t) = \int k\sqrt{t}\,dt = \frac{2k}{3}t^{3/2} + c$. So

$$P(0) = 500 = \frac{2k}{3}(0) + c = c.$$

So

$$P(t) = 500 = \frac{2k}{3}t^{3/2} + 500.$$

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And

$$P(1) = 600 = \frac{2k}{3}(1)^{3/2} + 500 \Rightarrow \frac{2k}{3} = 100.$$

So

$$P(t) = 100t^{3/2} + 500.$$

7. a) F increasing means $F' = f > 0$: on $(-3, 3.5)$. F decreasing means $F' = f < 0$: on $(3.5, 4)$.

b) F has a local max means $F' = f$ changes from $+$ to $-$: at $x = 3.5$. F has a local min means $F' = f$ changes from $-$ to $+$: never.

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c) F concave up means $F'' = f' > 0$, i.e., f is increasing: $(-1.5, 1.5)$. F concave down means $F'' = f' < 0$, i.e., f is decreasing: $(-3, -1.5)$ and $(1.5, 4)$.

d) F has a points of inflection when $F'' = f'$ changes sign, i.e., when f changes from increasing to decreasing or *vice versa*: at $x = \pm 1.5$.

g) The two graphs are parallel (differ only by a constant).