Math 131 Day 5

#1 $\int_2^3 (x^2-4)\,dx$

Use Right(n): $\Delta x = \frac{b-a}{n} = \frac{3-2}{n} = \frac{1}{n}$

Right endpt $\longrightarrow x_k = a + k\Delta x = 2 + \frac{k}{n}$

$f(x_k) = f(2+\frac{k}{n}) = 4 + \frac{4k}{n} + \frac{k^2}{n^2} - 4 = \frac{4k}{n} + \frac{k^2}{n^2}$

$$\text{Right}(n) = \sum_{k=1}^{n} \left(\frac{4k}{n} + \frac{k^2}{n^2}\right)\left(\frac{1}{n}\right) = \frac{4}{n^2}\sum_{k=1}^{n} k + \frac{1}{n^3}\sum_{k=1}^{n} k^2$$

5 $$= \frac{4}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \frac{2n+2}{n} + \frac{2n^2+3n+1}{6n^2} = 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$$

So $$\int_2^3 (x^2-4)\,dx = \lim_{n\to\infty} \text{Right}(n) = \lim_{n\to\infty} 7/3 + \frac{2}{n} + \frac{1}{2n} + \frac{1}{6n^2} = \boxed{\frac{7}{3}}$$

#2 a) $f(x) = x^2 + x$ on $[0,2]$ $\quad f'(x) = 2x+2 > 0$ on $[0,2]$

So $f$ is increasing on $[0,2]$. So Upper(n) = Right(n)

6 and Lower(n) = Left(n) since $f$ is increasing

b) Upper(n) = Right(n). $\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

right endpt $\longrightarrow x_k = a + k\Delta x = 0 + \frac{2k}{n} = \frac{2k}{n}$

$f(x_k) = \left(\frac{2k}{n}\right)^2 + \left(\frac{2k}{n}\right) = \frac{4k^2}{n^2} + \frac{2k}{n}$

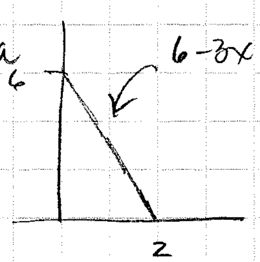
$$\text{Upper}(n) = \sum_{k=1}^{n}\left(\frac{4k^2}{n^2} + \frac{2k}{n}\right)\left(\frac{2}{n}\right) = \frac{8}{n^3}\sum_{k=1}^{n} k^2 + \frac{4}{n^2}\sum_{k=1}^{n} k$$

$$= \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{4}{n^2}\,\frac{n(n+1)}{2}$$

$$= \frac{4(2n^2+3n+1)}{3n^2} + \frac{2(n+1)}{n} = \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} + 2 + \frac{2}{n}$$

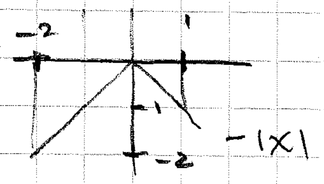
$$\int_0^2 x^2+2x\,dx = \lim_{n\to\infty} \text{Upper}(n) = \lim_{n\to\infty} \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} + 2 + \frac{2}{n} = \boxed{\frac{14}{3}}$$

#3 a

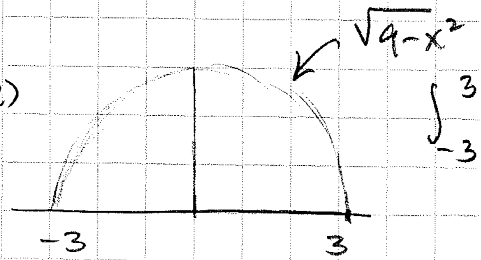



$$\int_0^2 6-3x\,dx = \text{Area of triangle} = \tfrac{1}{2}(2)(6) = 6$$

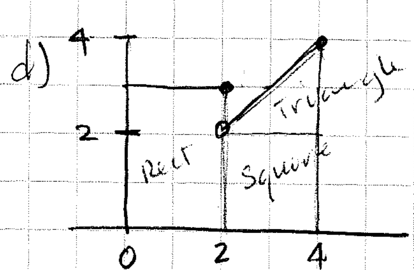
4

#3b) 


$$\int_{-2}^{1} -|x|\,dx = \text{net area of triangles}$$
$$= \tfrac{1}{2}(2)(-2) + \tfrac{1}{2}(1)(-1) = -2\tfrac{1}{2}$$

c) 


$$\int_{-3}^{3} \sqrt{9-x^2}\,dx = \text{net area of semi-circle}$$
$$= \frac{9}{2}\pi$$

d) 


$$\int_{0}^{4} f(x)\,dx = \text{net area}$$

Rect, Square, Triangle

$$= 2(3) + 2(2) + \tfrac{1}{2}(2)(2)$$
$$= 12$$

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#4 a) #36 $\displaystyle\int_{0}^{3\pi/2} x\sin x\,dx = \int_{0}^{\pi/2} x\sin x\,dx + \int_{\pi/2}^{\pi} x\sin x\,dx + \int_{\pi}^{3\pi/2} x\sin x\,dx$

2 $\quad = 1 + (\pi-1) - (\pi+1)$

$= -1$ below axis... negative area

#38 $\displaystyle\int_{\pi/2}^{2\pi} x\sin x\,dx = \int_{\pi/2}^{\pi} x\sin x\,dx + \int_{\pi}^{3\pi/2} x\sin x\,dx + \int_{3\pi/2}^{2\pi} x\sin x\,dx$

below x-axis

2 $\quad = (\pi-1) - (\pi+1) - (2\pi-1)$

$= -2\pi - 1$

b) #40 a) $\displaystyle\int_{1}^{4} -3f(x)\,dx = -3\int_{1}^{4} f(x)\,dx = -3(8) = -24$

2 c) $\displaystyle\int_{6}^{4} 12 f(x)\,dx = -12\int_{4}^{6} f(x)\,dx$

$\displaystyle = -12\left[\int_{1}^{6} f(x)\,dx - \int_{1}^{4} f(x)\,dx\right]$

$= -12[5-8] = \boxed{36}$

c) #44 a) $\displaystyle\int_{0}^{\pi/2} (2\sin\theta - \cos\theta)\,d\theta = -\int_{0}^{\pi/2} \cos\theta - 2\sin\theta\,d\theta = -(-1) = 1$

4 b) $\displaystyle\int_{\pi/2}^{0} (4\cos\theta - 8\sin\theta)\,d\theta = -4\int_{0}^{\pi/2} (\cos\theta - 2\sin\theta)\,d\theta = -4(-1) = 4$

#5 a) $\displaystyle\int \sqrt{7x}\,dx = \frac{2}{3}\frac{(7x)^{3/2}}{7} + C = \frac{2}{21}(7x)^{3/2} + C$ ← !

2 or $\displaystyle\int \sqrt{7}\,x^{1/2}\,dx = \frac{2\sqrt{7}}{3}x^{3/2} + C$

(b) $\displaystyle\int \sin\frac{\pi x}{3}\,dx$

$\displaystyle = -\frac{3}{\pi}\cos\left(\frac{\pi x}{3}\right) + C$

2

6. This problem uses the online applet `xFunctions`. There's a link to this applet at the course webpage or use `http://math.hws.edu/~mitchell/Math131F10/xFunctions.html` See the image below.

a) What is the area of a semi-circle of radius 1? Give your answer in terms of $\pi$ and also as a decimal to four places. $\frac{\pi}{2} \approx 1.5708$

b) The equation of the semi-circle of radius 1 centered at the origin is $f(x) = \sqrt{1-x^2}$ on the interval $[-1, 1]$ (see figure). We should be able to find the area of this region using calculus. According to our theory, since $f(x) = \sqrt{1-x^2}$ is continuous, it is integrable so

$$\text{Area} = \int_{-1}^{1} \sqrt{1-x^2}\,dx = \lim_{n\to\infty} \text{Right}(n) = \lim_{n\to\infty} \text{Left}(n).$$

So we should be able to approximate the answer using left and right Riemann sums with increasingly large values of $n$. Use `xFunctions` to find: Left(5), Right(5), and Midpoint(5). Then Left(52), Right(52), and Midpoint(52). Finally Left(512), Right(512), and Midpoint(512). Correctly round to four decimal places. Note: $\sqrt{1-x^2}$ is typed as `sqrt(1-x^2)`.

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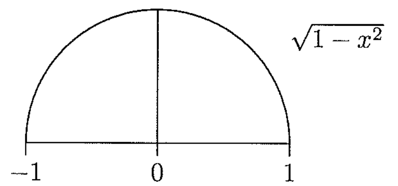


| $n$ | Left($n$) | Right($n$) | Midpoint($n$) |
|---|---|---|---|
| 5 | 1.4238 | 1.4238 | 1.6132 |
| 52 | 1.5664 | 1.5664 | 1.5721 |
| 512 | 1.5707 | 1.5707 | 1.5708 |

c) Are these estimates getting closer to your answer in part (a) as $n$ gets larger?