Math 131 Homework: Day 7

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2-6, 7–1pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

Review 5.3 as needed. Begin reading 5.4 on average values and the Mean Value Theorem for Integrals.

1. a) Practice is important. Page 346ff. Try #9, 11, 13 and 15.
b) Using FTC I: Page 347-8 #51–56 and 89. (Even Answers: e^x, $-\frac{2x}{x^2+1}$, $-\frac{1}{x^2+1}$.)
c) Working with definite integrals: Page 348 #75 (simplify first), 77, 79, and 81.
d) Working with definite integrals (assigned last time): Page 346–7 #25 (multiply out first), 27, 33–39 (odd), 41, 43, and 47. Remember, *net area* is signed area, so area below the axis is negative. Area is always positive, so area below the axis counts as positive area (you need to change its sign).

Homework

Complete the take-home quiz and bring it to Lab. Do WeBWorK set `Day07` (due Saturday night), also hand in the problems below on Friday. There are some similarities in the the two sets.

1. Review: This problem asks you to compute a definite integral two different ways: using Riemann sums and using the FTC. Review the Homework I handed back. The answers are on line.

a) Determine and simplify the formula for Right(n) for the function $f(x) = x^2 - x$ on the interval $[1, 4]$. Do this on another sheet and attach it to this one. Put your final formula below: $\Delta x = \frac{3}{n}$, $x_k = 1 + \frac{3k}{n}$

Right(n) = $\sum_{k=1}^{n}\left[\left(1+\frac{3k}{n}\right)^2 - \left(1+\frac{3k}{n}\right)\right]\frac{3}{n} = \sum_{k=1}^{n}\left[\left(1+\frac{6k}{n}+\frac{9k^2}{n^2}\right)-1-\frac{3k}{n}\right]\frac{3}{n} = \frac{3}{n}\sum_{k=1}^{n} \frac{3k}{n} + \frac{9k^2}{n^2}$

$\frac{9}{n^2}\sum_{k=1}^{n} k + \frac{27}{n^3}\sum k^2 = \frac{9}{n^2}\frac{n(n+1)}{2} + \frac{27}{n^3}\frac{(n(n+1)(2n+1))}{6}$

b) Determine the value of $\int_1^4 (x^2 - x)\,dx$ by using a limit of Riemann sums. Use correct limit notation.

$$\text{Right}(n) = \frac{9}{2n}(n+1) + \frac{9}{2n^2}(2n^2+3n+1) = \frac{9}{2} + \frac{9}{2n} + 9 + \frac{27}{2n} + \frac{9}{2n^2}$$

$$\int_1^4 (x^2-x)\,dx = \lim_{n\to\infty} \text{Right}(n) = \frac{9}{2} + 9 = 27/2$$

c) Using the Fundamental Theorem of Calculus, quickly evaluate $\int_1^4 (x^2 - x)\,dx$. (Are the answers the same?)

$$\int_1^4 (x^2-x)\,dx = \frac{x^3}{3} - \frac{x^2}{2}\Big|_1^4 = \frac{64}{3} - 8 - \left(\frac{1}{3} - \frac{1}{2}\right)$$

$$= 21 - 7\tfrac{1}{2} = 27/2$$

2. a) Page 332 #34. Be careful, net area is signed area. Show your work using properties of the integral.

$$\int_0^c f(x)\,dx = \int_0^a f(x)\,dx + \int_a^b f(x)\,dx + \int_b^c f(x)\,dx = 16 - 5 + 11 = 22$$

b) Look at the diagram on page 332 for #35–38. Use it to determine $\int_{2\pi}^{0} x \sin x\,dx$. Be careful of signs. Show your work using properties of the integral.

$$\int_{2\pi}^{0} x\sin x\,dx = -\int_0^{2\pi} x\sin x\,dx = -\left[\int_0^{\pi/2} x\sin x\,dx + \int_{\pi/2}^{\pi} x\sin x\,dx + \int_{\pi}^{3\pi/2} x\sin x\,dx + \int_{3\pi/2}^{2\pi} x\sin x\,dx\right.$$

$$= -\left[1 + (\pi - 1) - (\pi+1) - (2\pi - 1)\right]$$

3. Page 332 #42 (b only). Indicate your work. See WeBWorK Day07 for more like this.

$$\int_0^5 |f(x)|\,dx = \int_0^2 |f(x)|\,dx + \int_2^5 |f(x)|\,dx = |6| + |-8| = 14$$

4. Use the FTC (which part) to evaluate the following. Show your work.

a) $\int_1^2 \left(\frac{2}{s} - \frac{4}{s^2}\right) ds =$ $2\ln s + \frac{4}{s}\Big|_1^2 = 2\ln 2 + 2 - (0+4) = 2\ln 2 - 2$

b) $\int_0^{2\pi} \sec\frac{x}{8}\tan\frac{x}{8}\,dx =$ $8\sec\frac{x}{8}\Big|_0^{2\pi} = 8\sec\frac{2\pi}{8} - 8\sec 0$

$= 8\sec\frac{\pi}{4} - 8(1)$

$= 8\sqrt{2} - 8$

5. Use the FTC (which part) to simplify the following. Show your work. (See Example 5, p. 341.)

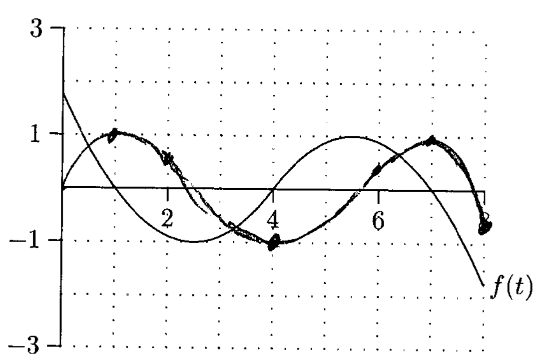
a) $\frac{d}{dx}\left[\int_x^{12} \cos(t^3)\,dt\right] =$ $\frac{d}{dx}\left[-\int_{12}^{x} \cos(t^3)\,dt\right] = -\cos(x^3)$

$u = \sin x,\ \frac{du}{dx} = \cos x$

b) $\frac{d}{dx}\left[\int_0^{\sin x} \frac{1}{1+t^6}\,dt\right] =$ $\frac{d}{du}\left[\int_0^u \frac{1}{1+t^6}\right]\frac{du}{dx} = \frac{1}{1+u^6}\cdot\frac{du}{dx}$

$= \frac{1}{1+(\sin x)^6}\cdot\cos x$

6. This is just like the earlier graphing problems you did on Lab. Review if necessary. Let $A(x) = \int_0^x f(t)\,dt$, where $f(t)$ is the function graphed below. $A(x)$ is the *net area* between f and the axis on the interval from 0 to the endpoint x. Use this relationship to answer the following questions.



a) Use the graph to estimate $A(0) =$ 0 $A(2) =$ 1/2

$A(4) =$ -1 $A(6) =$ 1/2 and $A(8) =$ -1/2

b) On what interval(s) is A increasing? Explain briefly.

(0, 1) and (4, 7)

where $A'(x) = f(x)$ is positive

c) At what point(s), if any, does A have a local max?

$x = 1, 7$ ← f switches from + to −

What about mins?

$x = 4$ − to +

Now make a rough sketch of the graph of $A(x)$ on the same axes using your values of A including maxs and mins.