Math 131 Homework: Day 8

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–1pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice and Reading

1. a) Reread and review Section 5.4 on average values, Read Section 5.5 on Substitution. This is very important and we will discuss it on Monday.

b) Average values: Page 354 #19, 21, 23, 25.

c) Read about definite integrals of odd and even functions (pages 349–350). Then do page 354–55 #7, 9, 13, and 39.

d) MVTI: Page 355 #31. First find f_{ave} and the point c where $f(c) = f_{\text{ave}}$.

Short Hand In for Monday and WeBWorK Day08 (due Monday night)

1. Do Lab 3, Problem 9(a). We want change in volume, previous work on lab3

$$V(3)-V(1) = \int_1^3 \frac{1}{3t+1}\,dt = \frac{1}{3}\int_1^3 \frac{3}{3t+1}\,dt = \frac{1}{3}\ln(3t+1)\Big|_1^3$$

$$= \frac{1}{3}\ln(10) - \frac{1}{3}\ln(4)$$

4

$$= \frac{1}{3}\ln\left(\frac{10}{4}\right)$$

$$= \frac{1}{3}\ln(5/2)$$

$$(\approx .03053)$$

2. Use the FTC to find $F'(x)$ if $F(x) = \int_{x^4}^2 8\sin(\pi t^2)\,dt$. Note the limits!

$u = x^4 \qquad \frac{du}{dx} = 4x^3$

$$F'(x) = \frac{d}{dx}\left[-\int_2^{x^4} 8\sin(\pi t^2)\,dt\right] = -8\sin(\pi(x^4)^2)\cdot 4x^3$$

$$= -8\sin(\pi x^8)\cdot 4x^3 = -32x^3\sin(\pi x^8)$$

3. Suppose that $\int_{1/2}^x g(t)\,dt = x^2\ln x$. Evaluate $g(1)$ and explain your answer. Hint: Apply FTC.

$$\frac{d}{dx}\left[\int_{1/2}^x g(t)\,dt\right] = \frac{d}{dx}\left[x^2\ln x\right] = 2\ln x + \frac{x^2}{x} = 2\ln x + x$$

So $g(x) = 2\ln x + x$ and $g(1) = 2\ln 1 + 1 = 1$

4. a) Breathing is cyclic. From the beginning of inhalation to the end of exhalation takes about 4 s. The **flow rate** of air into the lungs is modeled by $f(t) = \frac{1}{2}\sin(\frac{\pi}{2}t)$ liters/s. Find the **average** flow rate on the interval $[2, 4]$ seconds.

$$\text{Ave Val} = \frac{1}{b-a}\int_a^b f(t)\,dt = \frac{1}{4-2}\int_2^4 \frac{1}{2}\sin\left(\frac{\pi t}{2}\right)dt = -\frac{1}{4}\cdot\frac{2}{\pi}\cos\left(\frac{\pi t}{2}\right)\Big|_2^4$$

$$= -\frac{1}{2\pi}\left[\cos(2\pi) - \cos(\pi)\right]$$

$$= -\frac{1}{2\pi}\left[1-(-1)\right]$$

$$\boxed{= -\frac{1}{\pi}\text{ liters/s}}$$

b) **Extra credit**. The flow rate $f(t)$ is the rate of change in the volume $V(t)$ of air in the lungs. Find the **net change in the volume** of air in the lungs from time $t = 2$ to $t = 4$. What is going on physically during this period?

We want net change in $V(t)$ on $[2,4]$

$$V(4) - V(2) = \int_2^4 \frac{1}{2}\sin\left(\frac{\pi t}{2}\right) dt = -\frac{1}{2}\cdot\frac{2}{\pi}\cos\left(\frac{\pi t}{2}\right)\Big|_2^4$$

$$= -\frac{1}{\pi}\left[\cos(2\pi) - \cos(\pi)\right]$$

$$= -\frac{1}{\pi}\left[1 - (-1)\right] = \boxed{-\frac{2}{\pi} \text{ liters}}$$

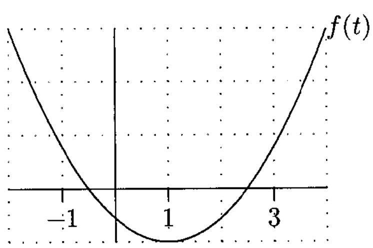
Exhaling!

5. OK, the FTC says that if $A(x) = \int_{-2}^{x} f(t)\,dt$, then $A'(x) = f(x)$. But also remember $A(x)$ is just the net area between f and the x axis on the interval from -2 to endpoint x.

a) At what point(s), if any, does A have a local max? Where $A' = f$ switches from + to − at $x = -1/2$

b) On what interval(s) is A decreasing? Explain briefly.

where $f = A' < 0$: $(-1/2, 2\,1/2)$



c) Is $A(0)$ a positive number or negative? Explain.

$A(0) = \int_{-2}^{0} f(t)\,dt$ = net area under f is positive ... more area above x-axis than below

d) Define $B(x) = \int_3^x f(t)\,dt$. Is $B(0)$ a positive number or negative? Explain. Think about net area and the limits of the integral.

$B(0) = \int_3^0 f(t)\,dt = -\int_0^3 f(t)\,dt = -$[net area from 0 to 3] $= -$[negative net area] = positive!

6. Page 355 #36. First find f_{ave} and then the point c where $f(c) = f_{\text{ave}}$. Give both the exact value of c and a decimal approximation.

$$f_{\text{ave}} = \bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-1}\int_1^4 \frac{1}{x}\,dx = \frac{1}{3}\ln|x|\Big|_1^4 = \frac{1}{3}\ln 4$$

$$\approx .4621$$

Need c so that

$$f(c) = \frac{1}{c} = \bar{f} \Rightarrow \frac{1}{c} = \frac{1}{3}\ln 4 \Rightarrow c = \frac{3}{\ln 4} \approx 2.164$$

same

7. Determine $\frac{d}{dx}\left[\int_1^{x^3} \ln(t^2+1)\,dt + \int_{x^3}^{100} \ln(t^2+1)\,dt\right] = \frac{d}{dx}\left[\int_1^{x^3} \ln(t^2+1)\,dt - \int_{100}^{x^3} \ln(t^2+1)\right]$

$= 0 \ldots$