Math 131 Day 20

#1) f $\int \frac{\ln x}{x}\,dx = \int u\,du = \frac{u^2}{2}+c = \frac{1}{2}\ln^2 x + c$

$u=\ln x,\ du=\frac{1}{x}\,dx$

h) $\int (x^2+1)e^x\,dx = \overset{u}{(x^2+1)}\overset{v}{e^x} - \overset{-v\,du}{\int 2xe^x dx} = (x^2+1) - [2xe^x - \int 2e^x dx]$

$u=x^2+1 \quad dv=e^x dx \qquad u=2x \quad dv=e^x dx$

$du=2x\,dx \quad v=e^x \qquad du=2dx \quad v=e^x$

$= (x^2+1)e^x - 2xe^x + 2e^x = (x^2-2x+3)e^x + c$

m) $\int \frac{1}{\sqrt{1-9x^2}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{3}\arcsin u + c = \frac{1}{3}\arcsin(3x)+c$

$u^2=9x^2 \quad u=3x \quad du=3dx,\ \frac{1}{3}du=dx$

#2 p458 #12

$\int se^{-2s}\,ds = -\frac{1}{2}se^{-2s} - \int -\frac{1}{2}e^{-2s}dx = -\frac{1}{2}se^{-2s} - \frac{1}{4}e^{-2s} + c$

$u=s \quad dv=e^{-2s}dx$

$du=ds \quad v=-\frac{1}{2}e^{-2s}$

or use substition $w=x^2$, $dw=2x\,dx$, $\frac{1}{2}dw = x\,dx$

$\frac{1}{2}\int \tan(w)\,dw$

p458 #22

$\int x\tan^{-1}(x^2)\,dx = \frac{1}{2}x^2\arctan(x^2) - \int \frac{x^3}{1+x^4}dx$

$u=1+x^4$

$du=4x^3dx$

$\frac{1}{4}du = x^3dx$

$u=\arctan(x^2) \quad dv = x\,dx$

$du=\frac{2x}{1+x^4}dx \quad v=\frac{1}{2}x^2$

$= \frac{1}{2}x^2\arctan(x^2) - \int \frac{1}{4}\frac{1}{u}du$

$= \frac{1}{2}x^2\arctan(x^2) - \frac{1}{4}\ln|u| + c$

$= \frac{1}{2}x^2\arctan(x^2) - \frac{1}{4}\ln|1+x^4| + c$

p458 #14 $\int \theta\sec^2\theta\,d\theta = \theta\tan\theta - \int \tan\theta\,d\theta$

$u=\theta \quad dv=\sec^2\theta\,d\theta$

$du=d\theta \quad v=\tan\theta$

$= \theta\tan\theta - \ln|\sec\theta| + c$

p458 #26

$$\int x^2 \ln^2 x\,dx = \frac{x^3}{3}\ln^2 x - \int \frac{2x^2}{3}\ln x\,dx$$

$u = \ln^2 x$ $\qquad dv = x^2\,dx$

$du = \frac{2\ln x}{x}\,dx$ $\qquad V = \frac{x^3}{3}$

$u = \ln x$ $\qquad dv = \frac{2x^2}{3}\,dx$

$du = \frac{1}{x}\,dx$ $\qquad V = \frac{2}{9}x^3$

$$= \frac{x^3}{3}\ln^2 x - \left[\frac{2}{9}x^3\ln x - \int \frac{2}{9}x^2\,dx\right]$$

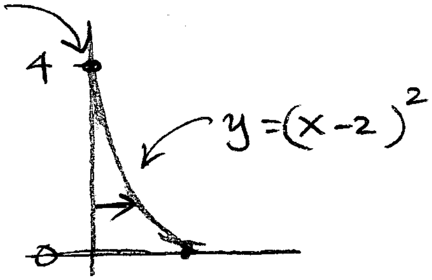
$$= \frac{x^3}{3}\ln^2 x - \frac{2}{9}x^3\ln x + \frac{2}{27}x^3 + C$$

#3 Work $\quad y = (x-2)^2 \Rightarrow \sqrt{y} = x-2 \Rightarrow x = \sqrt{y}+2$

$A(y) = \pi x^2 = \pi(\sqrt{y}+2)$

$= \pi(y + 4\sqrt{4} + 4)$

Top of tank @ $x=0$ ... $y=(0-2)^2=4$



Top layer of liquid

a) $W = \int_0^4 \pi(y+4\sqrt{y}+4)[4-y]\,dy$

bot of liquid $\quad$ move to

2 feet above top

b) $W = \int_0^4 \pi(y+4\sqrt{y}+4)[6-y]\,dy$

top of oil $\quad$ move to top of tank

c) $W = \int_0^1 \pi(y+4\sqrt{y}+4)[4-y]\,dy$