Math 131 — Day 21

#1 $\int e^x\cos(7x)dx = e^x\cos(7x) + \int e^x 7\sin(7x)dx$!

$u=\cos 7x \quad dv = e^x dx$ $\qquad u = 7\sin(7x) \quad dv = e^x dx$

$du = -7\sin(7x)dx \quad v = e^x$ $\qquad du = 49\cos(7x)dx \quad v = e^x$

So $\int e^x\cos(7x)dx = e^x\cos(7x) + 7e^x\sin(7x) - 49\int e^x\cos(7x)dx$

6 $\quad 50\int e^x\cos(7x)dx = e^x\cos(7x) + 7e^x\sin(7x)$

Finally: $\int e^x\cos(7x)\,dx = \dfrac{e^x[\cos(7x)+7\sin(7x)]}{50} + C$

#2 $\int_0^{1/\sqrt{2}} y\arctan y^2\,dy$ $\qquad u=y^2 \quad du = 2y\,dy \quad \tfrac12 du = y\,dy$ $\qquad y=0 \Rightarrow u=0 \quad y=1/\sqrt2 \Rightarrow u=1/2$

6 $\quad = \frac12\int_0^{1/2}\arctan u\,du = \frac12\left[u\arctan u\Big|_0^{1/2} - \int_0^{1/2}\frac{u}{1+u^2}du\right]$ easy sub

$w = \arctan u \quad dv = du$

$dw = \frac{1}{1+u^2}du \quad v = u$

$= \frac12\left[\left(\tfrac12\arctan\tfrac12 - 0\right) - \tfrac12\ln(1+u^2)\Big|_0^{1/2}\right]$

$= \frac12\left[\tfrac12\arctan\tfrac12 - \tfrac12\ln 5/4\right] \approx .0601$

#3 Shells $\int_0^\pi 2\pi x\sin x\,dx = -2\pi x\cos x + \int_0^\pi 2\pi\cos x\,dx$

6 $\quad u = 2\pi x \quad dv = \sin x\,dx$

$du = 2\pi dx \quad v = -\cos x$

$= -2\pi x\cos x + 2\pi\sin x\Big|_0^\pi$

$= [-2\pi^2(-1)+0] - [0+0]$

$= 2\pi^2$

#4) $\int\cos x\ln(\sin x)dx = \int\ln y\,dy$

5 $\quad y = \sin x \quad dy = \cos x\,dx$ $\qquad u = \ln y \quad dv = dy$ $\quad du = \frac1y dy \quad v = y$

$= y\ln y - \int 1\,dy = y\ln y - y + C$

$= \sin x\ln(\sin x) - \sin x + C$

As above

#5a) $\int x\ln x^2\,dx = \frac12\int\ln y\,dy = \frac12[y\ln y - y] + C$

5 $\quad y = x^2 \quad dy = 2x\,dx \Rightarrow \tfrac12 dy = x\,dx$

$= \frac12[x^2\ln x^2 - x^2] + C$

5 b) By Parts

$$\int x \ln x^2 \, dx = \frac{x^2}{2} \ln x^2 - \int \frac{2}{x} \cdot \frac{x^2}{2} \, dx$$

5 $u = \ln x^2 \qquad dv = x\,dx$

$du = \frac{2x}{x^2}\,dx = \frac{2}{x}\,dx \qquad v = \frac{x^2}{2}$

$$= \frac{x^2}{2} \ln x^2 - \int x \, dx$$

$$= \frac{x^2}{2} \ln x^2 - \frac{x^2}{2} + C$$

#6 a) $\int \sin^2(3x)\,dx = \int \frac{1}{2} - \frac{1}{2}\cos(6x)\,dx$

4 $= \frac{1}{2}x - \frac{1}{12}\sin(6x) + C$

b) $\int \cos^4 x \, dx = \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int \cos^2 x \, dx$

Reduction Formula

Reduce Again

5 $= \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\left[\frac{1}{2}\cos x \sin x + \frac{1}{2}\underbrace{\int \cos^0 x \, dx}_{\int 1\,dx}\right]$

$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}\cos x \sin x + \frac{3}{8}x + C$

OR

$\int \cos^4 x \, dx = \frac{1}{4}\cos^3 x \sin x + 3/4 \int \cos^2 x \, dx$

$= \frac{1}{4}\cos^3 x \sin x + 3/4 \int \frac{1}{2} + \frac{1}{2}\cos(2x)\,dx$

$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}x + \frac{3}{16}\sin(2x) + C$

#7 $\int \sin\sqrt{x}\,dx = \int 2y \sin y \, dy = -2y\cos y + \int 2\cos y \, dy$

$y = \sqrt{x}$

$dy = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,dy = dx$

$2y\,dy = dx$

$u = 2y \qquad dv = \sin y \, dy$

$du = 2\,dy \qquad v = -\cos y$

$= -2y\cos y + 2\sin y + C$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$