# Math 131 Day 22

#1

| $\theta$ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | 1 |
| $\cos\theta$ | 1 | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | 0 |

$\cos^2(3\theta) = \frac{1}{2} + \frac{1}{2}\cos(6\theta)$

$\sin^2(4\theta) = \frac{1}{2} - \frac{1}{2}\cos(8\theta)$

$\cos^2(-2\theta) = \frac{1}{2} + \frac{1}{2}\cos(-4\theta)$

#2a $\int \sin^2(4x)\,dx = \int \frac{1}{2} - \frac{1}{2}\cos(8x)\,dx = \frac{1}{2}x - \frac{1}{16}\sin(8x) + C$

b) $\int \cos^2(\pi x)\,dx = \int \frac{1}{2} + \frac{1}{2}\cos(2\pi x) = \frac{1}{2}x + \frac{1}{4\pi}\sin(2\pi x) + C$

c) $\int \cos^5(x)\,dx = \frac{\cos^4(x)\sin x}{5} + \frac{4}{5}\int \cos^3(x)\,dx$

$= \frac{\cos^4(x)\sin x}{5} + \frac{4}{5}\left[\frac{\cos^2 x \sin x}{3} + \frac{2}{3}\int \cos x\,dx\right]$

$= \frac{\cos^4(x)\sin x}{5} + \frac{4\cos^2 x \sin x}{15} + \frac{8\sin x}{15} + C$

d) $\int \sin^3 2x \cos^4 2x\,dx = \int \sin^2 2x \cos^4 2x \cdot \sin 2x\,dx$

$= \int (1-\cos^2 2x)\cos^4 2x \cdot \sin 2x\,dx$

$u = \cos 2x$

$du = -2\sin 2x$

$-\frac{1}{2}du = \sin 2x$

$= -\frac{1}{2}\int (1-u^2)(u^4)\,du$

$= \frac{1}{2}\int u^6 - u^4\,du = \frac{u^7}{14} - \frac{u^5}{10} + C = \frac{\cos^7 2x}{14} - \frac{\cos^5 2x}{10} + C$

e) $\int \tan^5 x\,dx = \frac{1}{4}\tan^4 x - \int \tan^3 x\,dx$

$= \frac{1}{4}\tan^4 x - \left[\frac{1}{2}\tan^2 x - \int \tan x\,dx\right]$

$= \frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \ln|\sec x| + C$