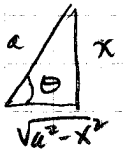
Math 131 Day 24

#1 $\int \sqrt{a^2-x^2}\,dx$



$\frac{x}{a} = \sin\theta \Rightarrow \theta = \arcsin\left(\frac{x}{a}\right)$

$x = a\sin\theta$

$\sqrt{a^2-x^2} = a\cos\theta$

$dx = a\cos\theta\,d\theta$

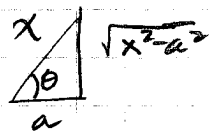
$= \int a\cos\theta\, a\cos\theta\,d\theta = \int a^2\cos^2\theta\,d\theta = a^2\left[\frac{1}{2}\cos\theta\sin\theta + \frac{1}{2}\int d\theta\right]$

your value of a^2 will vary

$= a^2\left[\frac{1}{2}\cos\theta\sin\theta + \frac{1}{2}\theta\right] + C = \frac{a^2}{2}\left[\frac{\sqrt{a^2-x^2}}{a}\cdot\frac{x}{a} + \arcsin\left(\frac{x}{a}\right)\right]$

$= \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right) + C$

#2 $\int \frac{1}{x^2\sqrt{x^2-a^2}}\,dx$



$\frac{x}{a} = \sec\theta$

$x = a\sec\theta$

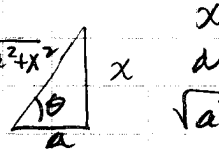
$\sqrt{x^2-a^2} = a\tan\theta$

$dx = a\sec\theta\tan\theta\,d\theta$

$= \int \frac{a\sec\theta\tan\theta\,d\theta}{a^2\sec^2\theta\, a\tan\theta} = \frac{1}{a^2}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{a^2}\int \cos\theta\,d\theta = \frac{1}{a^2}\sin\theta + C$

$= \frac{1}{a^2}\frac{\sqrt{x^2-a^2}}{x} + C$

#3 $\int_0^a \frac{1}{\sqrt{x^2+a^2}}\,dx$



$x = a\tan\theta$ $\qquad \sin\theta = \frac{x}{\sqrt{x^2+a^2}}$

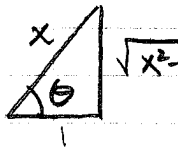
$dx = a\sec^2\theta\,d\theta$

$\sqrt{a^2+x^2} = a\sec\theta \Rightarrow (x^2+a^2)^{3/2} = a^3\sec^3\theta$

$= \int \frac{a\sec^2\theta\,d\theta}{a^3\sec^3\theta} = \frac{1}{a^2}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{a^2}\int \cos\theta\,d\theta = \frac{1}{a^2}\sin\theta + C$

$= \frac{1}{a^2}\cdot\frac{x}{\sqrt{x^2+a^2}}\Big|_0^a = \frac{1}{a^2}\left[\frac{a}{\sqrt{2a^2}} - 0\right] = \frac{1}{\sqrt{2}a^2}$

#4 p473 #38 $\int \frac{1}{x^3\sqrt{x^2-1}}\,dx$



$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = +\tan\theta \Rightarrow \theta = \arctan\left(\sqrt{x^2-1}\right)$

$= \int \frac{\sec\theta\tan\theta}{\sec^3\theta\tan\theta}\,d\theta = \int \frac{1}{\sec^2\theta}\,d\theta = \int \cos^2\theta\,d\theta$

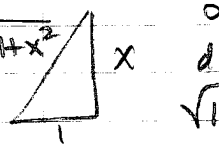
$= \frac{1}{2}\cos\theta\sin\theta + \frac{1}{2}\int d\theta = \frac{1}{2}\cos\theta\sin\theta + \frac{1}{2}\theta + C$

$= \frac{1}{2}\left[\frac{1}{x}\cdot\frac{\sqrt{x^2-1}}{x} + \arctan\left(\sqrt{x^2-1}\right)\right] + C$

$= \frac{1}{2}\left[\frac{\sqrt{x^2-1}}{x^2} + \arctan\left(\sqrt{x^2-1}\right)\right] + C$

Math 181 Day 24

#5 p 473 #30 $\int \frac{x^4}{1+x^2}\,dx$

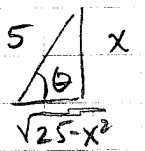


$x = \tan\theta$
$dx = \sec^2\theta\,d\theta$
$\sqrt{1+x^2} = \sec\theta$

$$= \int \frac{\tan^4\theta}{\sec^2\theta}\sec^2\theta\,d\theta = \int \tan^4\theta\,d\theta = \frac{1}{3}\tan^3\theta - \int \tan^2\theta\,d\theta$$

$$= \frac{1}{3}\tan^3\theta - \int \sec^2\theta - 1\,d\theta = \frac{1}{3}\tan^3\theta - \tan\theta + \theta + C$$
$$= \frac{x^3}{3} - x + \arctan x + C$$

#6 $\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

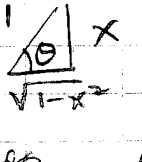


$x = 5\sin\theta \rightsquigarrow \sin\theta = x/5$
$dx = 5\cos\theta\,d\theta$
$\theta = \arcsin\left(\frac{x}{5}\right)$
$\sqrt{25-x^2} = 5\cos\theta$

$$= \int \frac{25\sin^2\theta\,5\cos\theta\,d\theta}{5\cos\theta} = 25\int \sin^2\theta\,d\theta$$

$$= 25\left[\frac{-\sin\theta\cos\theta}{2} + \frac{1}{2}\int d\theta\right] = 25\left[\frac{-\sin\theta\cos\theta}{2} + \frac{1}{2}\theta\right] = \cdots$$
$$= \frac{25}{2}\left[-\frac{x}{5}\frac{\sqrt{25-x^2}}{5} + \arcsin\left(\frac{x}{5}\right)\right] + C = \frac{-x\sqrt{25-x^2}}{2} + \frac{25}{2}\arcsin\left(\frac{x}{5}\right) + C$$

#7 $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$



$x = \sin\theta$
$dx = \cos\theta\,d\theta$
$\sqrt{1-x^2} = \cos\theta$

$$= \int \frac{\sin^3\theta\cos\theta}{\cos\theta}\,d\theta = \int \sin^3\theta\,d\theta = -\frac{1}{3}\sin^2\theta\cos\theta + \frac{2}{3}\int \sin\theta\,d\theta$$
$$= -\frac{1}{3}\sin^2\theta\cos\theta - \frac{2}{3}\cos\theta + C = -\frac{1}{3}x^2\sqrt{1-x^2} - \frac{2}{3}\sqrt{1-x^2} + C$$

#8 $\int x\cos^2 x\,dx = \int \frac{x}{2} + \frac{x}{2}\cos(2x)\,dx = \frac{x^2}{4} + \int \frac{x}{2}\cos 2x\,dx$

$u = \frac{x}{2}$ $\quad dv = \cos(2x)\,dx$
$du = \frac{1}{2}dx$ $\quad v = \frac{1}{2}\sin(2x)$

$$= \frac{x^2}{4} + \frac{x}{4}\sin(2x) - \int \frac{1}{4}\sin(2x)\,dx$$

$$= \frac{1}{4}\left[x^2 + x\sin(2x) + \frac{1}{2}\cos(2x)\right] + C$$