Math 131 Homework Day 25

My Office Hours: M & W 12:30–2:00, Tu 2:30–4:00, & F 1:15-2:30 or by appointment. **Math Intern** Sun: 12–6pm; M 3-10pm; Tu 2–6, 7–1pm; W and Th: 5-10 pm in Lansing 310. Website: `http://math.hws.edu/~mitchell/Math131S13/index.html`.

Practice

After one more triangle substitution today, we will start on our final technique of integration: partial fractions.

1. Read and review Section 7.3 and the handout on trig substitution.

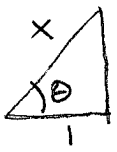
2. Reread Section 7.4 through Examples 1 and 2 through page 480. You should skim the rest of the section, but we will not cover it in any detail. Also see the online notes for today.
   a) Try page 483 #9, 11, 13, 15, and if we get this far: #19, 21, and 23.
   b) **What's ahead**: We will skip to Section 7.8 on Improper Integrals. This will require using L'Hôpital's rule to evaluate certain limits. You should be familiar with L'Hôpital's rule from Calculus I. Review Section 4.7 (page 280). There are online notes, too—way back from Day 1. Take a look!

Hand In At Lab, Use this sheet.

THESE ARE NOT EASY. Use your handout and notes to help set these up. You will have to use your trig integral skills to do the actual integrations.

0. Start WeBWorK `Day25` due Sunday night. Finish `Day24` due Thursday night.

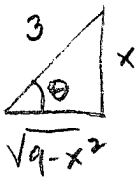
1. Setting up. True or False (show your work for this): $\int \frac{\sqrt{x^2-1}}{x}\,dx = \int \sec\theta\tan\theta\,d\theta$ · False



$x = \sec\theta$
$dx = \sec\theta\tan\theta$
$\sqrt{x^2-1} = \tan\theta$

$= \int \frac{\tan\theta \sec\theta \tan\theta\,d\theta}{\sec\theta} = \int \tan^2\theta\,d\theta$

2. Try $\int x^3\sqrt{9-x^2}\,dx$. What trig integration technique must you use?



$x = 3\sin\theta$
$dx = 3\cos\theta\,d\theta$
$\sqrt{9-x^2} = 3\cos\theta$

$\int 27\sin^3\theta\, 3\cos\theta\, 3\cos\theta\,d\theta = 243\int \sin^3\theta\cos^2\theta\,d\theta$ ← odd

$= 243\int \sin^2\theta\cos^2\theta\sin\theta\,d\theta$

$= 243\int (1-\cos^2\theta)\cos^2\theta\sin\theta\,d\theta$

$= -243\int (1-u^2)u^2\,du$

$u = \cos\theta$
$du = -\sin\theta\,d\theta$
$-du = \sin\theta$

$= -243\int u^2-u^4\,du = -243\left[\frac{u^3}{3}-\frac{u^5}{5}\right]+C$

$= -243\left[\frac{\cos^3\theta}{3}-\frac{\cos^5\theta}{5}\right]+C$

$= -243\left[\frac{(9-x^2)^{3/2}}{81}-\frac{(9-x^2)^{5/2}}{5(243)}\right]+C$

$= \frac{(9-x^2)^{5/2}}{5} - 3(9-x^2)^{3/2}+C$

OVER ☞

3. Try these three problems.

a) $\int \frac{2x+2}{x^2-4}\,dx$ b) $\int \frac{1}{x^2+3x+2}\,dx$ c) $\int \frac{6x}{(x+2)(x^2-1)}\,dx$

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a)

$$\frac{2x+2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{Ax+2A+Bx-2B}{(x-2)(x+2)}$$

x: $2 = A+B$

const: $2 = 2A-2B$

$A = 2A+2B$ } Add

$6 = 4A$

$A = 3/2$, $B = 1/2$

$$\int \frac{2x+2}{x^2-4}\,dx = \int \frac{3/2}{x-2} + \frac{1/2}{x+2}\,dx$$

$$= \frac{3}{2}\ln|x-2| + \frac{1}{2}\ln|x+2| + C$$

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b)

$$\frac{1}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2} = \frac{Ax+2A+Bx+B}{(x+1)(x+2)}$$

x: $0 = A+B$

const: $1 = 2A+B$ ← subtract

$1 = A$

$-1 = B$

$$\int \frac{1}{x^2+3x+2}\,dx = \int \frac{1}{x+1} - \frac{1}{x+2}\,dx$$

$$= \ln|x+1| - \ln|x+2| + C$$

$$= \ln\left|\frac{x+1}{x+2}\right| + C$$

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c) $\frac{6x}{x+2(x-1)(x+1)} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{x+1} = Ax^2 - A + Bx^2+3Bx+2B+Cx^2+Cx-2C$

x^2: $0 = A+B+C$

x: $6 = 3B+C$

const: $0 = -A+2B-2C$ | Add

$6 = 6B$

$B = 1$

$C = 3$

$A = -4$

$$\int \frac{6x}{(x+2)(x^2-1)}\,dx = \int \frac{-4}{x+2} + \frac{1}{x-1} + \frac{3}{x+1}\,dx$$

$$= -4\ln|x+2| + \ln|x-1| + 3\ln|x+1| + C$$