Math 121 ~~Day~~ 27 — Use Correct Notation

#1a) $\lim\limits_{x\to 0^+} \overset{\nearrow 0}{x^2}\, \overset{\nearrow -\infty}{\ln x} = \lim\limits_{x\to 0^+} \dfrac{\overset{\nearrow -\infty}{\ln x}}{\underset{\searrow \infty}{1/x^2}} \overset{L'H}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{2}{x^3}} = \lim\limits_{x\to 0^+} \dfrac{-x^2}{2} = \boxed{0}$

b) $\lim\limits_{x\to\infty} \overset{\nearrow\infty}{x}\, \overset{\nearrow 0}{\tan(1/x)} = \lim\limits_{x\to\infty} \dfrac{\overset{\nearrow 0}{\tan(1/x)}}{\underset{\searrow 0}{1/x}} = \lim\limits_{x\to\infty} \dfrac{-1/x^2 \sec^2(1/x)}{-1/x^2}$

$= \lim\limits_{x\to\infty} \sec^2(1/x) = \sec 0 = \boxed{1}$

c) $\lim\limits_{x\to\infty} (1-2/x)^x = y \qquad 1^\infty$

$\ln y = \ln \lim\limits_{x\to\infty} (1-2/x)^x = \lim\limits_{x\to\infty} \overset{\nearrow\infty}{x}\, \overset{\nearrow 0}{\ln(1-2/x)} = \lim\limits_{x\to\infty} \dfrac{\ln(1-2/x)}{1/x}$

$\overset{L'H}{=} \lim\limits_{x\to\infty} \dfrac{\left(\frac{1}{1-2/x}\right)\left(\frac{2}{x^2}\right)}{-1/x^2} = \lim\limits_{x\to\infty} -\overset{\nearrow 1}{\left(\frac{1}{1-2/x}\right)}(2) = -2$

$\ln y = -2 \Rightarrow y = \boxed{e^{-2}} = \lim\limits_{x\to\infty} (1-2/x)^x$

d) $\lim\limits_{x\to 0} \dfrac{\overset{\nearrow 0}{\sin kx}}{\underset{\searrow 0}{\arcsin x}} = \lim\limits_{x\to 0} \dfrac{k\cos(kx)}{\frac{1}{\sqrt{1-x^2}}} = \dfrac{k}{1} = \boxed{k}$

e) $\lim\limits_{x\to 0^+} x^{3x} = y \qquad 0^0$

$\ln y = \ln \lim\limits_{x\to 0^+} x^{3x} = \lim\limits_{x\to 0^+} \overset{\nearrow 0}{3x}\, \overset{\nearrow -\infty}{\ln x} = \lim\limits_{x\to 0^+} \dfrac{\overset{\nearrow -\infty}{3\ln x}}{\underset{\searrow\infty}{1/x}}$

$\overset{L'H}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{3}{x}}{-1/x^2} = \lim\limits_{x\to 0^+} -3x = 0$

$\ln y = 0 \Rightarrow y = e^0 = \boxed{1 = \lim\limits_{x\to 0^+} x^{3x}}$

f) $\lim\limits_{x\to\infty} \ln(2x-2) - \ln(x+7) = \lim\limits_{x\to\infty} \ln\left(\overset{\nearrow 2}{\dfrac{2x-2}{x+7}}\right) = \ln 2$

#2 $\displaystyle\int_0^\infty \frac{1}{(x+1)^3}\,dx = \lim_{b\to\infty} \int_0^b (x+1)^{-3}\,dx = \lim_{b\to\infty} -\frac{1}{2}(x+1)^{-2}\Big|_0^b$

$= \lim\limits_{b\to\infty} -\dfrac{1}{2}\left[\underset{\to 0}{\dfrac{1}{(b+1)^2}} - \dfrac{1}{1^2}\right] = \dfrac{1}{2}$

#3 p 510 #8 USE Correct Notation

$$\Rightarrow \int_0^\infty \frac{1}{\sqrt[3]{x+2}}\,dx = \lim_{b\to\infty} \int_0^b (x+2)^{-1/3}\,dx = \lim_{b\to\infty} \frac{3}{2}(x+2)^{2/3}\Big|_0^b$$

$$= \lim_{b\to\infty} \frac{3}{2}\left[\overset{\nearrow\infty}{(b+2)^{2/3}} - 2^{2/3}\right] = \infty \text{ diverges}$$

#6 Bonus

$$\lim_{x\to0^+} \overset{0^0}{(\tan x)^x} = y$$

$$\ln y = \ln \lim_{x\to0^+} (\tan x)^x = \lim_{x\to0^+} \overset{\nearrow 0}{x}\,\overset{\nearrow -\infty}{\ln(\tan x)} = \lim_{x\to0^+} \frac{\ln(\tan x)}{1/x}\;\frac{-\infty}{+\infty}$$

$$\overset{LH}{=} \lim_{x\to0^+} \frac{\frac{\sec^2 x}{\tan x}}{-\frac{1}{x^2}} = \lim_{x\to0^+} -\frac{\overset{\nearrow0}{x^2\sec^2 x}}{\underset{\searrow0}{\tan x}} \overset{L'H}{=} \lim_{x\to0^+} \frac{2x\sec^2 x - x^2\sec^2 x\tan x}{\sec^2 x}$$

$$= \lim_{x\to0^+} \overset{\nearrow0}{2x} - x^2\overset{\nearrow0}{\tan x} = 0$$

$$\ln y = 0 \Rightarrow y = e^0 = \boxed{1 = \lim_{x\to0^+} (\tan x)^x}$$

#4

$$\int_0^\infty 2xe^{-x^2}\,dx = \lim_{b\to\infty} \int_0^b 2xe^{-x^2}\,dx = \lim_{b\to\infty} -e^{-x^2}\Big|_0^b$$

$u = -x^2$

$du = -2x\,dx$

$-du = 2x\,dx$

$$= \lim_{b\to\infty} -\underbrace{e^{-b^2}}_{\to 0} + e^0 = \boxed{1}$$

$\lim_{x\to\infty} e^{-x} = 0$

#5

$$\int_0^\infty \frac{dx}{1+x^2} = \lim_{b\to\infty} \int_0^b \frac{1}{1+x^2}\,dx = \lim_{b\to\infty} \arctan x\Big|_0^b$$

$$= \lim_{b\to\infty} \overset{\nearrow\pi/2}{\arctan b} - \arctan 0 = \frac{\pi}{2}$$