Day 29 Math 131

#1 a) $\int \frac{8}{x^2+2x-3}\,dx = \int \frac{2}{x-1} - \frac{2}{x+3}\,dx = 2\ln|x-1| - 2\ln|x+3| + C$

$= 2\ln\left|\frac{x-1}{x+3}\right| + C$

$\frac{8}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{Ax+3A+Bx-B}{(x-1)(x+3)}$

$x: 0 = A+B$

const: $8 = 3A - B$

$8 = 4A \quad A = 2,\ B = -2$

$\Longrightarrow$ Simplify to evaluate limits (Highest powers)

b) $\int_2^\infty \frac{8}{x^2+2x-3}\,dx = \lim_{b\to\infty} 2\ln\left|\frac{x-1}{x+3}\right|\Bigg|_2^b = \lim_{b\to\infty} 2\ln\left|\frac{b-1}{b+3}\right| - 2\ln\frac{1}{5}$ $\to 1$

$= 2\ln 1 + 2\ln 5 = \boxed{2\ln 5}$

c) $\int_0^{\boxed{1}} \frac{8}{x^2+2x-3}\,dx$ ← improper $= \lim_{b\to 1^-} 2\ln\left|\frac{x-1}{x+3}\right|\Bigg|_0^b$ ← as $x\to 0$, $\ln x \to -\infty$

$= \lim_{b\to 1^-} 2\ln\left|\frac{b-1}{b+3}\right| - 2\ln\frac{1}{3} = -\infty$ Diverges

$\to -\infty$

#2 a) $\int_1^\infty \frac{1}{x^{5/4}}\,dx = \frac{1}{\frac{5}{4}-1} = 4 \quad (p>1)$

b) $\int_1^\infty \frac{1}{x^{2/3}}\,dx$ diverges $(p \le 1)$

c) $\int_1^\infty \frac{2}{x^7}\,dx = 2\int_1^\infty \frac{1}{x^7}\,dx = \frac{2}{7-1} = \frac{1}{3} \quad (p>1)$

d) $\int_1^\infty \frac{1}{x^{-15}}\,dx$ Diverges $p = -15 \le 1$

#3 a) $\int \frac{4x^3}{(1+x^4)^2}\,dx = \int \frac{1}{u^2}\,du = -\frac{1}{u} = -\frac{1}{1+x^4} + C$

$u = 1+x^4,\ du = 4x^3\,dx$

b) $\int_{-\infty}^\infty \frac{4x^3}{(1+x^4)^2}\,dx = \int_{-\infty}^0 \frac{4x^3}{(1+x^4)^2}\,dx + \int_0^\infty \frac{4x^3}{(1+x^4)^2}\,dx = -1 + 1 = \boxed{0}$ ← See below

$\int_{-\infty}^0 \frac{x^3}{(1+x^4)^2}\,dx = \lim_{b\to-\infty} -\frac{1}{1+x^4}\Bigg|_b^0 = \lim_{b\to-\infty} -1 + \frac{1}{1+b^4} = -1+0 = -1$

and $\int_0^\infty \frac{x^3}{(1+x^4)^2}\,dx = \lim_{b\to+\infty} -\frac{1}{1+x^4}\Bigg|_0^b = \lim_{b\to\infty} -\frac{1}{1+b^4} + 1 = 0+1 = 1$

#4 a) p534 #10 $a_n = n + 1/n$: $\{2, 2\tfrac{1}{2}, 3\tfrac{1}{3}, 4\tfrac{1}{4}, \ldots\}$

b) #16 $a_{n+1} = a_n + a_{n-1}$; $a_1 = 1$, $a_0 = 1$,

$\{1, 1, 2, 3, 5, 8 \ldots\}$

c) $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots\}$; $a_n = \frac{1}{2n}$ $n = 1, 2, 3, \ldots$

d) $\{1/4, 2/9, 3/16, 4/25, \ldots\}$ $a_n = \frac{n}{(n+1)^2}$ $n = 1, 2, 3, \ldots$

e) $\{64, 32, 16, 8, 4, \ldots\}$ $a_1 = 64$; $a_{n+1} = \frac{1}{2} a_n$ $n = 1, 2, 3 \ldots$

#5 a) $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{\cancel{n}(n+1)(2n+1)}{9n^{\cancel{3}\,2}} = \lim\limits_{n\to\infty} \frac{2n^2+3n+1}{9n^2}$

$= \lim\limits_{n\to\infty} \frac{2}{9} + \cancel{\frac{1}{3n}}^{\to 0} + \cancel{\frac{1}{9n^2}}^{\to 0} = 2$

b) $\lim\limits_{n\to\infty} (1 + 4/n)^n = y = \boxed{e^4}$ $\quad 1^\infty$

$\ln y = \ln \lim\limits_{n\to\infty} (1+4/n)^n = \lim\limits_{n\to\infty} \overset{\nearrow\infty}{n} \overset{\nearrow 0}{\ln(1+4/n)}$

$= \lim\limits_{n\to\infty} \frac{\ln(1+4/n)}{1/n} \overset{L'H}{=} \lim\limits_{n\to\infty} \frac{\frac{1}{1+4/n} \cdot \frac{-4}{n^2}}{-1/n^2} = \lim\limits_{n\to\infty} \frac{4}{1+4/n} = 4$

so $\boxed{y = e^4}$

c) $\lim\limits_{n\to\infty} \overset{\nearrow\infty}{n} \overset{\nearrow 0}{\sin(1/n)} = \lim\limits_{n\to\infty} \frac{\overset{\nearrow 0}{\sin(1/n)}}{\underset{\searrow 0}{1/n}} \overset{L'H}{=} \lim\limits_{n\to\infty} \frac{\cos(1/n)(-1/n^2)}{-1/n^2}$

$= \lim\limits_{n\to\infty} \cos(1/n) = \cos(0) = \boxed{1}$

#6 a) $\int_1^3 \frac{x-3}{x^2+3x-4}\,dx = \lim\limits_{b\to 1^+} \int_b^3 \frac{x-3}{x^2+3x-4}\,dx$

$(x+4)(x-1)$

b) $\int_1^3 \frac{x-3}{x^2+4}\,dx$ Not improper

c) $\int_1^3 \frac{x-3}{x^2-4}\,dx = \lim\limits_{b\to 2^-} \int_1^b \frac{x-3}{x^2-4}\,dx + \lim\limits_{a\to 2^+} \int_a^3 \frac{x-3}{x^2-4}\,dx$

$(x-2)(x+2)$ ... improper at $x = 2$