Math 131 Day 31

Key Limits

#1 a)
$$\lim_{n\to\infty} (1-6/n)^{n/3} = \lim_{n\to\infty} \left[(1-6/n)^{n}\right]^{1/3} = \left[e^{-6}\right]^{1/3} = e^{-2}$$

b) $$\lim_{n\to\infty} n^{4/n} = \lim_{n\to\infty} (n^{1/n})^4 = 1^4 = 1$$

c) $$\lim_{n\to\infty} 4^n 7^{-n} = \lim_{n\to\infty} \left(\frac{4}{7}\right)^n = 0 \quad \leftarrow |r| = 4/7 < 1$$ Geometric seq

d) $$\lim_{n\to\infty} (-1)^{-n} = \lim_{n\to\infty} \left(\frac{1}{-1}\right)^n = \lim_{n\to\infty} (-1)^n \text{ DNE} \quad r=-1$$ Geometric seq

#2 $$\lim_{n\to\infty} \overbrace{(n+2)^{1/n}}^{\infty^0} = y = 1$$

$$\ln y = \lim_{n\to\infty} \ln (n+2)^{1/n} = \lim_{x\to\infty} \frac{\overbrace{\ln(n+2)}^{\to\infty}}{\underbrace{n}_{\to\infty}} \overset{L'H}{=} \lim_{n\to\infty} \frac{\frac{1}{n+2}\cdot 1}{1} = 0$$

$\ln y = 0$ so $y = e^0 = 1$

#3 a) $d_1 = 20$

$d_2 = 20 + \frac{1}{3}d_1 = 26\frac{2}{3}$

$d_3 = 20 + \frac{1}{3}d_2 = 28.888$

$d_4 = 20 + \frac{1}{3}d_3 = 29.63$

$\boxed{d_{n+1} = 20 + \frac{1}{3}d_n, \quad d_1 = 20}$

b) See above, yes non-decreasing

c) $$\lim_{n\to\infty} d_n = L = \lim_{n\to\infty} d_{n+1}$$

so $$\lim_{n\to\infty} d_{n+1} = \lim_{n\to\infty} 20 + \frac{1}{3}d_n$$

$$L = 20 + \frac{1}{3}L \Rightarrow \frac{2}{3}L = 20 \Rightarrow \boxed{L = 30}$$

4) a) $$\sum_{n=0}^{\infty} (5/7)^n = \frac{1}{1-5/7} = 7/2 \qquad (|r| = 5/7 < 1)$$

b) $$\sum_{n=0}^{\infty} 4(-2/5)^n = 4\cdot\left(\frac{1}{1-(-2/5)}\right) = 4\left(\frac{1}{7/5}\right) = \frac{20}{7} \qquad |r| = \frac{2}{5} < 1$$

c) $$\sum_{n=0}^{\infty} 6(5/3)^n$$ diverges b/c $|r| = \frac{5}{3} \geq 1$

\# 5 $\displaystyle\int_{\boxed{-2}}^{1} \frac{2}{x^2+5x+6}\,dx$ ← Improper at -2

$$\frac{2}{x^2+5x+6} = \frac{A}{x+2} + \frac{B}{x+3} = \frac{Ax+3A+Bx+2B}{x^2+5x+6}$$

x: $A+B=0$; $A=-B$

const: $3A+2B=2$; $A=2,\ B=-2$

$$= \lim_{a\to -2^+} \int_a^1 \frac{2}{x+2} - \frac{2}{x+3}\,dx = \lim_{a\to -2^+} 2\ln|x+2| - 2\ln|x+3| \Big|_a^1$$

$$= \lim_{a\to -2^+} 2\ln\left|\frac{x+2}{x+3}\right| \Big|_a^1 = \lim_{a\to -2^+} 2\ln\left|\frac{3}{4}\right| - 2\ln\left|\frac{a+2}{a+3}\right|$$

$\frac{a+2}{a+3} \to 0$

$$= 2\ln 3/4 - (-\infty)$$

DNE (diverges)

\# 6 $\displaystyle\sum_{k=0}^{\infty} \frac{2}{k^2+5k+6} = \sum_{k=0}^{\infty}\left(\frac{2}{k+2} - \frac{2}{k+3}\right)$

$$S_n = \sum_{k=0}^{n}\left(\frac{2}{k+2} - \frac{2}{k+3}\right) = \underbrace{\left(\frac{2}{2} - \cancel{\frac{2}{3}}\right)}_{k=0} + \underbrace{\left(\cancel{2/3} - \cancel{2/4}\right)}_{k=1} + \underbrace{\left(\cancel{2/4} - \cancel{2/5}\right)}_{k=2} + \cdots + \left(\frac{2}{n+2} - \frac{2}{n+3}\right)$$

$$S_n = \left(2/2 - 2/n+3\right)$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 1 - 2/n+3 = \underline{1}$$

So $\displaystyle\sum_{k=0}^{\infty} \frac{2}{k^2+5x+6} = \lim_{n\to\infty} S_n = 1$