Math 131 Day 32

#1a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n = \frac{a}{1-r} = \frac{1}{1-2/3} = \boxed{3}$

b) $\displaystyle\sum_{n=0}^{\infty} 4\left(-2/5\right)^n = \frac{a}{1-r} = \frac{4}{1-(-2/5)} = \boxed{20/7}$

c) $\displaystyle\sum_{n=0}^{\infty} 6\left(5/4\right)^n$ $\boxed{\text{Diverges}}$ $|r| = 5/4 \geq 1$

#2a) $\displaystyle\sum_{k=0}^{\infty}\left(4/3\right)^{-k} = \sum_{k=0}^{\infty}\left(3/4\right)^k = \frac{a}{1-r} = \frac{1}{1-3/4} = \boxed{4}$

b) $\displaystyle\sum_{n=0}^{\infty} 3\left(2/5\right)^{2n} = \underbrace{3}_{a=3} + \frac{12}{25} + \frac{48}{625} + \cdots = \frac{a}{1-r} = \frac{3}{1-4/25} = \frac{75}{21} = \boxed{\frac{25}{7}}$

$r = 4/25$

c) $\displaystyle\sum_{k=1}^{\infty} 4\left(1/3\right)^k = \underbrace{\frac{4}{3}}_{a} + \frac{4}{9} + \frac{4}{27} + \cdots = \frac{a}{1-r} = \frac{4/3}{1-1/3} = \boxed{2}$

$r = 1/3$

d) $\displaystyle\sum_{k=2}^{\infty} 3\left(-1/2\right)^k = \underbrace{\frac{3}{4}}_{a} - \frac{3}{8} + \frac{3}{16} + \cdots = \frac{a}{1-r} = \frac{3/4}{1-(-1/2)} = \boxed{\frac{1}{2}}$

$r = -1/2$

#3 $\displaystyle\sum_{k=0}^{\infty}\left(\frac{1}{k+2} - \frac{1}{k+4}\right)$ Dif = 2

$S_n = \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{1/5} - 1/7\right) + \left(\cancel{1/6} - 1/8\right) + \cdots$

$\cdots + \left(\cancel{\frac{1}{n}} - \cancel{\frac{1}{n+2}}\right) + \left(\frac{1}{n+1} - \frac{1}{n+3}\right) + \left(\cancel{\frac{1}{n+2}} - \frac{1}{n+4}\right)$

$S_n = 1/2 + 1/3 - 1/n+3 - 1/n+4$

$\displaystyle\lim_{n\to\infty} S_n = \lim_{n\to\infty} 1/2 + 1/3 - \frac{1}{n+3} - \frac{1}{n+4} = \frac{1}{2} + \frac{1}{3} = \boxed{5/6}$

#4 $\displaystyle\sum_{k=0}^{\infty} \ln\left(\frac{k+2}{k+1}\right) = \sum_{k=0}^{\infty} \ln(k+2) - \ln(k+1)$ Diff = 1

$S_n = (\cancel{\ln 2} - \ln 1) + (\cancel{\ln 3} - \cancel{\ln 2}) + (\ln 3 - \ln 4) + \cdots$

$+ [\cancel{\ln n} - \ln(n-1)] + [\cancel{\ln(n+1)} - \cancel{\ln n}] + [\ln(n+2) - \cancel{\ln(n+1)}]$

$S_n = \ln(n+2) - \ln 1 = \ln(n+2)$

$\displaystyle\lim_{n\to\infty} S_n = \lim_{n\to\infty} \ln(n+2) = \infty \ldots$ $\boxed{\text{Diverges}}$

#5 $\sum_{k=1}^{\infty} [\arctan(k+1) - \arctan k]$ DIFF = 1

$S_n = (\arctan 2 - \arctan 1) + (\arctan 3 - \arctan 2) + \cdots$

$\cdots + (\arctan n - \arctan(n-1)) + (\arctan(n+1) - \arctan n)$

$S_n = \arctan(n+1) - \arctan 1$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \arctan(n+1) - \arctan 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

#6 $8 + 6 + \frac{9}{2} + \frac{27}{8} + \frac{81}{32} + \cdots$ $a=8$, $r = 3/4$ $\quad \frac{a}{1-r} = \frac{8}{1-3/4} = 32$

#7 a) $\sum_{k=0}^{\infty} \left[\frac{1}{2}(.2)^k + \frac{3}{2}(.8)^k\right] = \sum_{k=0}^{\infty} \frac{1}{2}(.2)^k + \sum_{k=0}^{\infty} \frac{3}{2}(.8)^k = \frac{1/2}{1-.2} + \frac{3/2}{1-.8}$

split into two pieces as w/ Riemann sums

$= .625 + 7.5 = \boxed{8.125}$

b) $\sum_{k=0}^{\infty} \frac{2-3^k}{6^k} = \sum_{k=0}^{\infty} 2\left(\frac{1}{6}\right)^k - \sum_{k=0}^{\infty} \left(\frac{3}{6}\right)^k = \frac{2}{1-1/6} - \frac{1}{1-1/2} = \frac{12}{5} - 2 = \boxed{\frac{2}{5}}$

$(\frac{1}{2})^k$

#8 XC Divergence Test:

$\sum_{k=1}^{\infty} \frac{k}{k^2+1} \cdots \lim_{n\to\infty} \frac{n}{n^2+1} \overset{HP}{=} \lim_{n\to\infty} \frac{n}{n^2} = 0$ Div Test Inconclusive

$\sum \frac{k^2}{2^k} \qquad \lim_{n\to\infty} \frac{n^2}{2^n} = \lim_{x\to\infty} \frac{x^2}{2^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{\ln 2 \cdot 2^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2}{(\ln 2)^2 2^x} = 0$

Divergence Test is Inconclusive

$\sum_{k=1}^{\infty} \frac{k^3}{k^3+1} \cdots \lim_{n\to\infty} \frac{n^3}{n^3+1} \overset{HP}{=} 1 \neq 0 \therefore$ The series diverges