Math 131 Day 3#3

#1 a) $\lim_{n\to\infty} \frac{3n+1}{2n+5} = \lim_{n\to\infty} \frac{3+1/n}{2+5/n} = \frac{3}{2} \neq 0$. So by the nth term test for divergence, the series $\sum_{n=1}^{\infty} \frac{3n+1}{2n+5}$ diverges

b) $\lim_{n\to\infty} \frac{n}{2n^2+1} = \lim_{n\to\infty} \frac{1/n}{2+1/n^2} = 0$. So the nth term test <u>does not apply</u>

c) $\lim_{n\to\infty} (1.1)^n = \infty \neq 0$ (key limit $|r|>1$). So by the nth term test $\sum_{n=1}^{\infty} (1.1)^n$ diverges

d) $\lim_{n\to\infty} \left(1+\frac{4}{n}\right)^n = e^4 \neq 0$ (key limit). So by the nth term test, $\sum_{n=1}^{\infty} (1+4/n)^n$ diverges

e) $\lim_{n\to\infty} \sqrt[n]{n} = 1 \neq 0$. By the nth term test, $\sum_{n=1}^{\infty} \sqrt[n]{n}$ diverge

f) $\lim_{n\to\infty} \frac{n!}{n^n} = 0$. The nth term test for divergence does not apply.

#2 a) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges by the p-series test since $p = 1/2 \leq 1$

b) $\sum_{n=1}^{\infty} \frac{1}{e^{2n}} = \sum_{n=1}^{\infty} \left(\frac{1}{e^2}\right)^n$ Rewrite converges by the geometric series test since $|r| = \frac{1}{e^2} < 1$

c) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by the p-series test since $p = 3 > 1$

d) $\sum_{n=1}^{\infty} \frac{1}{9+n^2}$ ... integral test .... $\frac{1}{9+x^2}$ is positive, decreasing continuous

$$\int_1^{\infty} \frac{1}{9+x^2}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{\underset{\uparrow a^2}{9}+x^2}\,dx = \lim_{b\to\infty} \frac{1}{3}\arctan\left(\frac{x}{3}\right)\Big|_1^b$$

$$= \lim_{b\to\infty} \frac{1}{3}\arctan\left(\frac{b}{3}\right) - \frac{1}{3}\arctan(1/3) = \frac{1}{3}\left(\frac{\pi}{2}\right) - \frac{1}{3}\arctan\left(\frac{1}{3}\right)$$

Converges. By the integral test $\sum_{n=1}^{\infty} \frac{1}{9+n^2}$ also converges

#2e) $\sum_{n=1}^{\infty} \frac{3}{n^2+7n+10}$ ... Integral Test: $f(x)=\frac{3}{x^2+7x+10}$ is cont, pos for $x\geq 1$ and decreasing ... the numerator is constant and $x^2+7x+10$ increases as x increases, so the whole function decreases

$$\int_1^{\infty} \frac{3}{x^2+7x+10}dx = \lim_{b\to\infty} \int_1^b \frac{1}{x+2}-\frac{1}{x+5}dx = \lim_{b\to\infty} \ln\left|\frac{x+2}{x+5}\right| \Big|_1^b$$

$$= \lim_{b\to\infty} \ln\left|\frac{b+2}{b+5}\right| - \ln 3/6 = \ln 1 + \ln 2 = \ln 2$$

Since the integral converges so does $\sum \frac{3}{n^2+7n+10}$ by Integral Test

2f) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^3}} = \sum \frac{3}{n^{3/2}}$ Converges by p-series test $(p=3/2>1)$

2g) $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ ... $f(x)=\frac{x}{2x^2+1}$; $f'(x)=\frac{2x^2+1-x(4x)}{(2x^2+1)^2} = \frac{1-2x^2}{(2x^2+1)^2}<0$ for $x\geq 1$

So f is positive, cont, decreasing ... apply integral test

$$\int_1^{\infty}\frac{x}{2x^2+1} = \lim_{b\to\infty}\int_1^b \frac{x}{2x^2+1}dx = \lim_{b\to\infty}\frac{1}{4}\ln|2x^2+1|\Big|_1^b$$

$$= \lim_{b\to\infty}\frac{1}{4}\ln(\underbrace{2b^2+1}_{\to\infty}) - \ln 3 = \infty \text{ Diverges}$$

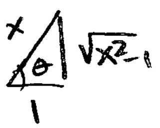
So by the Integral Test $\sum_{n=1}^{\infty}\frac{n}{2n^2+1}$ also diverges

2h) $\sum_{n=1}^{\infty} \frac{1}{n^{1.00001}}$ converges by the p-series test $(p=1.00001>1)$

2i) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$ ... Integral test $f(x)=\frac{1}{\sqrt{x^2-1}}$ is pos & cont for $x\geq 2$

$f'(x) = -1/2(x^2-1)^{-3/2}(2x)<0$ for $x\geq 2$ (Decr)

$$\int_2^{\infty}\frac{1}{\sqrt{x^2-1}}dx = \lim_{b\to\infty}\ln|x+\sqrt{x^2-1}|\Big|_2^b = \lim_{b\to\infty}\ln|\underbrace{b+\sqrt{b^2-1}}_{\to\infty}| - \ln|2+\sqrt{3}|$$



$x=\sec\theta$
$dx=\sec\theta\tan\theta\,d\theta$
$\sqrt{x^2-1}=\tan\theta$

$$\int\frac{1}{\sqrt{x^2-1}}dx = \int\frac{\sec\theta\cancel{\tan\theta}\,d\theta}{\cancel{\tan\theta}} = \ln|\sec\theta+\tan\theta|+c$$
$$=\ln|x+\sqrt{x^2-1}|+c$$

$= \infty$ Diverges

Since $\int_2^{\infty}\frac{1}{\sqrt{x^2-1}}dx$ diverges so does $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n^2-1}}$ by the integral test.