Math 131 Day 35

#1 $\sum_{k=1}^{\infty} \frac{k^2}{2^k}$ Apply Divergence Test

$$\lim_{k\to\infty} \frac{k^2}{2^k} = \lim_{x\to\infty} \frac{x^2}{2^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{2^x \ln 2} \overset{L'H}{=} \lim_{x\to\infty} \frac{2}{\underbrace{2^x(\ln 2)^2}_{\to\infty}} = 0$$

The test is inconclusive

Note

#2 $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ Integral test: $f(x) = \frac{1}{x(\ln x)^2}$ $x \geq 2$ is cont (since x and $\ln x$ are) and positive and decreasing since the numerator is constant and the denominator gets larger as x increases

$$\text{so } \int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx = \lim_{b\to\infty} \left.\frac{-1}{\ln x}\right|_2^b = \lim_{b\to\infty} \underbrace{\frac{-1}{\ln b}}_{\to 0} + \frac{1}{\ln 2} = \frac{1}{\ln 2}$$

$u = \ln x$
$du = 1/x$

$$\int \frac{1}{u^2}\,du = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$$

Since the integral converges, so does $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ by the Integral Test

#3 a) $\sum \frac{1}{k^{3/2}+1}$ limit comparison with $\sum \frac{1}{k^{3/2}}$. The terms are pos.

$$\lim_{k\to\infty} \frac{1}{k^{3/2}+1} \cdot \frac{k^{3/2}}{1} \overset{HP}{=} 1 \text{ and } 0<1<\infty.$$

Since $\sum \frac{1}{k^{3/2}}$ converges by the p-series test ($p = 3/2 > 1$), so does $\sum \frac{1}{k^{3/2}+1}$ by the limit comparison test

b) $\sum_{k=1}^{\infty} \frac{1}{3k-\sqrt{k}}$ compare to $\sum \frac{1}{k}$. The terms are positive

and $\lim_{k\to\infty} \frac{1}{3k-\sqrt{k}} \cdot \frac{k}{1} \overset{HP}{=} \lim_{k\to\infty} \frac{k}{3k} = 1/3$. $0 < 1/3 < \infty$

Since $\sum 1/k$ diverges (p-series $p=1$) So does $\sum \frac{1}{3k-\sqrt{k}}$ by limit Comparison

c) $\sum \sin(1/k)$ ← terms are positive since $0 < 1/k < \pi$

Compare to $\sum 1/k$

$$\lim_{k\to\infty} \frac{\sin(1/k)}{1/k} = \lim_{x\to\infty} \frac{\sin(1/x)}{1/x} \overset{L'H}{=} \lim_{x\to\infty} \frac{\cancel{-1/x^2} \cos(1/x)}{\cancel{-1/x^2}} = \cos(0) = 1$$

Since $0<1<\infty$ and since $\sum 1/k$ diverges (p-series, $p=1$) then by limit comparison $\sum \sin(1/k)$ also diverges

#4a) $\sum_{K=1}^{\infty} \frac{2^k}{k!}$ Ratio Test ... terms are positive

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$$

The series converges by the ratio test

b) $\sum_{K=1}^{\infty} \frac{2^k}{k^k}$ Ratio Test ... terms are positive

Same power

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n} = \lim_{n\to\infty} \frac{2 \cdot n^n}{(n+1)^{n+1}} = \lim_{n\to\infty} \frac{2}{n+1} \cdot \frac{n^n}{(n+1)^n}$$

$$= \lim_{n\to\infty} \frac{2}{n+1} \cdot \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \frac{2}{n+1} \left(\frac{1}{1+1/n}\right)^n = 0 \cdot \frac{1}{e} = 0 < 1$$

By the ratio test, the series converges

c) $\sum_{n=1}^{\infty} \frac{k!}{9^k}$ The terms are positive ... Ratio Test

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)!}{9^{n+1}} \cdot \frac{9^n}{n!} = \lim_{n\to\infty} \frac{n+1}{9} = \infty > 1$$

By the ratio test, the series Diverges

#5) $\sum_{K=2}^{\infty} \frac{5\ln k}{k}$ The terms are positive and $0 < \frac{1}{k} < \frac{5\ln k}{k}$ for all k

Since $\sum_{k=2}^{\infty} 1/k$ diverges, by Direct comparison, $\sum_{k=2}^{\infty} \frac{5\ln k}{k}$

also diverges

#6 $\sum \frac{k^2}{2^k}$ The terms are positive ... Ratio Test

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^2 \cdot \frac{1}{2} = \frac{1}{2} < 1$$

By the ratio test, $\sum \frac{k^2}{2^k}$ converges.

Bonus problem – Hand in Wed Does $\sum_{K=1}^{\infty} \frac{k+1}{k^k}$ converge?

Justify your answer?