0. Determine whether the following series converge. Justify your answers with an ARGUMENT:

a) EZ: $\displaystyle\sum_{n=1}^{\infty} \frac{5n^7 - 11n}{2n^8 + 11n^2}$ ~ Limit Comparison w/ $\sum \frac{1}{n}$; The terms $\frac{5n^7-11n}{2n^8+11n^2}$ and $\frac{1}{n}$ are both positive (once $n \geq 2$), so

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{5n^7-11n}{2n^8+11n^2} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{5n^8 - 11n^2}{2n^8+11n^2} \overset{HP}{=} \frac{5}{2}$$

Since $0 < L = 5/2 < \infty$, by the limit comparison test since $\sum 1/n$ diverges (p-series, p=1) so does $\sum \frac{5n^7-11n}{2n^8+11n^2}$.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(n!)^2}$ Factorial... Try Ratio Test. The terms are positive and

$$r = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{4^{n+1}}{((n+1)!)^2} \cdot \frac{(n!)^2}{4^n} = \lim_{n\to\infty} \frac{4}{(n+1)^2} = 0 < 1$$

Since $r < 1$, by the Ratio Test the series converges

c) Does $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{\cancel{k}\,k}k^2}{\cancel{\sqrt{k^6+1}}\ k^3+1}$ converge conditionally, absolutely or not at all. (Simplify first) Alternating

$a_k = k^2/k^3+1 > 0$. Check 2 conditions 1. $\displaystyle\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{k^2}{k^3+1} \overset{HP}{=} \lim_{k\to\infty} \frac{1}{k} = 0$ ✓

2. Decr? $f(x) = \frac{x^2}{x^3+1}$, $f'(x) = \frac{2x(x^3+1) - x^2(3x^2)}{(x^3+1)^2} = \frac{1-x^3}{(x^3+1)^3} \leq 0$ ✓ $(x \geq 1)$

By the alternating series test, the series converges

Check Abs convergence

$\sum \left| \frac{(-1)^k k^2}{k^3+1} \right| = \sum \frac{k^2}{k^3+1}$. Use limit comparison w/ $\sum 1/k$. positive ✓

$$L = \lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{k^2}{k^3+1} \cdot \frac{k}{1} = \lim_{k\to\infty} \frac{k^3}{k^3+1} \overset{HP}{=} 1.$$

so $0 < L < \infty$; since $\sum \frac{1}{k}$ diverges (p-series, p=1) so does $\sum \left| \frac{(-1)^k k^2}{k^3+1} \right|$ by Limit comparison

Conditionally convergent

d) Does $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)\arctan n}{n^3}$ converge conditionally, absolutely or not at all. (Simplify first)

Try absolute convergence first. ~ $\sum \frac{1}{n^3}$

$\sum \left| \frac{\cos(n\pi)\arctan n}{n^3} \right| = \sum_{n=1}^{\infty} \frac{\arctan n}{n^3}$. Both series have positive terms. Limit comparison:

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\arctan n}{n^3} \cdot \frac{n^3}{1} = \lim_{n\to\infty} \arctan n = \frac{\pi}{2}. \text{ So } 0 < L = \frac{\pi}{2} < \infty$$

Since $\sum \frac{1}{n^3}$ converges (p-series, $p = 3 > 1$), so does $\sum \left| \frac{\cos(\pi n)\arctan n}{n^3} \right|$. STOP

ABSOLUTELY CONV!)