Math 131 Day 39

#1 $f(x) = \ln(1-x)$ — Find $p_4(x)$ @ $a=0$

$f(x) = \ln(1-x)$ $\quad f(0) = \ln 1 = 0$

$f'(x) = -(1-x)^{-1}$ $\quad f'(0) = -1$

$f''(x) = -(1-x)^{-2}$ $\quad f''(0) = -1$

$f'''(x) = -2(1-x)^{-3}$ $\quad f'''(0) = -2$

$f^{(4)}(x) = -3!(1-x)^{-4}$ $\quad f^{(4)}(0) = -3!$

$$p_4(x) = f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3 + \frac{f^{(4)}(0)}{4!}(x-0)^4$$

$$p_4(x) = 0 - (x-0) - \tfrac{1}{2}(x-0)^2 - \tfrac{1}{3}(x-0)^3 - \tfrac{1}{4}(x-0)^4$$

$$= -x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4$$

$$p_n(x) = \sum_{k=1}^{n} \frac{-x^k}{k}$$

#2 $\sum \frac{(n+2)!}{(-9)^{n+1}}$ check Abs Conv $\sum \left|\frac{(n+2)!}{(-9)^{n+1}}\right| = \sum \frac{(n+2)!}{9^{n+1}}$

Use Ratio Test Extension

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+3)!}{(-9)^{n+2}} \cdot \frac{(-9)^{n+1}}{(n+2)!}\right| = \lim_{n\to\infty}\left|\frac{n+3}{-9}\right| = \infty > 1$$

By the ratio test extension the series diverges

#3 a $\sum \frac{(-1)^k x^k}{5^k}$. Radius of Convergence

$$r = \lim_{k\to\infty} \sqrt[k]{\left|\frac{(-1)^k x^k}{5^k}\right|} = \left|\frac{x}{5}\right| < 1 \Rightarrow |x| < 5$$

To converge

Radius of Conv: $r = 5$

b) Interval: Check end pts $|x| = 5 \Rightarrow x = \pm 5$

at $x=5$: $\sum (-1)^k \frac{5^k}{5^k} = \sum (-1)^k$ Diverges (Geometric series $|r| = 1$)

combine

at $x=-5$ $\sum (-1)^k \frac{(-5)^k}{5^k} = \sum \frac{5^k}{5^k} = \sum 1$ Diverges by n term test $\lim_{n\to\infty} 1 = 1 \neq 0$

Interval of Conv: $(-5, 5)$

#4) $\sum 5n(x-4)^n$ Radius of conv:

$$r = \lim_{n\to\infty}\left|\frac{(5n+5)(x-4)^{n+1}}{5n\ (x-4)^n}\right| = \lim_{n\to\infty}\left|(1+1/n)(x-4)\right| = |x-4| \underset{\text{To converge}}{<} 1$$

r = rad of conv = 1

Interval of convergence: Check endpts $|x-4|=1 \Rightarrow x=5, 3$

@ $x=5$

$\sum 5n(5-4)^n = \sum 5n$ Diverges by n^{th} term test: $\lim_{n\to\infty} 5n = \infty \neq 0$

at $x=3$

$\sum 5n(3-4)^n = \sum(-1)^n 5n$. Diverges by n^{th} term test

$\lim_{n\to\infty}(-1)^n 5n$ DNE.

Interval: $(3,5)$

#5 $\sum \frac{(-1)^n x^n}{2n}$ Ratio Test

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{n+1}}{2n+2}\cdot\frac{2n}{(-1)^n x^n}\right| = \lim_{n\to\infty}\left|\frac{2n}{2n+2}\cdot x\right| = |x| \underset{\text{to converge}}{<} 1$$

radius of conv = 1

Interval of convergence $|x|=1 \Rightarrow x = \pm 1$

At $x=1$ $\sum_{n=1}^{\infty}\frac{(-1)^n(1)^n}{2n} = \sum\frac{(-1)^n}{2n}$. Alt. Series Test $a_n = 1/2n > 0$ ✓

1. $\lim_{n\to\infty} a_n = \lim_{n\to\infty} 1/2n = 0$ ✓
2. Decreasing – $\frac{1}{2(n+1)} \leq \frac{1}{2n}$ ∴ Decreasing ✓

} The series converges at $x=1$ by Alt Ser Test

At $x=-1$: $\sum\frac{(-1)^n(-1)^n}{2n} = \sum_{n=1}^{\infty}\frac{1}{2n} = \frac{1}{2}\sum\frac{1}{n}$ Diverges... (p-series, p=1)

Interval of Convergence: $(-1, 1]$ ← included

#6 $\sum\frac{3^n x^n}{n!}$ ... use ratio test

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{3^{n+1}x^{n+1}}{(n+1)!}\cdot\frac{n!}{3^n x^n}\right| = \lim_{n\to\infty}\left|\frac{3x}{n+1}\right| = 0 \underset{\text{Always true}}{<} 1$$

The series converges for all x: $r=\infty$; Interval $(-\infty, \infty)$