Math 131 Day 41

#1a) $\sum\left(-\frac{x}{10}\right)^{2k}$ Radius/Interval of Conv — Root Test

$$\lim_{k\to\infty}\sqrt[k]{|A_k|}=\lim_{k\to\infty}\sqrt[k]{\left|-\frac{x}{10}\right|^{2k}}=\left|\frac{x}{10}\right|^2=\left|\frac{x^2}{100}\right|<1\Rightarrow |x|^2<100\Rightarrow \underset{R}{|x|<10}$$

Endpts $x=\pm 10$

At $x=-10$: $\sum\left(-\frac{(-10)}{10}\right)^{2k}=\sum\left[(-1)^{2}\right]^{k}=\sum(1)^k$ ... Diverges by n^{th} term test $\lim_{n\to\infty}a_n=1\neq 0$

At $x=10$: $\sum\left(-\frac{10}{10}\right)^{2k}=\sum(-1)^{2k}=\sum(1)^k$ SAME " diverges

Interval: $(-10,10)$

b) $\sum\frac{(x-3)^{2n}}{4^n n}$ ... Radius/Interval of Conv — Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{(x-3)^{2n+2}}{4^{n+1}(n+1)}\cdot\frac{4^n n}{(x-3)^{2n}}\right|=\lim_{n\to\infty}\left|\frac{(x-3)^2}{4}\cdot\frac{n}{n+1}\right|=\left|\frac{(x-3)^2}{4}\right|<1$$

so $|(x-3)^2|<4\Rightarrow |x-3|<2=R$ ... Endpts $x=\underset{3+2}{5},\underset{3-2}{1}$

At $x=1$: $\sum\frac{(1-3)^{2n}}{4^n n}=\sum\frac{(-2)^{2n}}{4^n n}=\sum\frac{\left[(-2)^2\right]^n}{4^n n}=\sum\frac{4^n}{4^n n}=\sum\frac{1}{n}$

diverges (p-series $p=1$)

At $x=5$: $\sum\frac{(5-3)^{2n}}{4^n n}=\sum\frac{\left[2^2\right]^n}{4^n n}=\sum\frac{1}{n}$ diverges (p series, $p=1$)

Interval: $(1,5)$

#2

$f(x)=\cos 2x$	$f(0)=1=2^0$
$f'(x)=-2\sin 2x$	$f'(0)=0$
$f''(x)=-2^2\cos 2x$	$f''(0)=-(2^2)$
$f'''(x)=2^3\sin 2x$	$f'''(0)=0$
$f^{(4)}(x)=2^4\cos(2x)$	$f^{(4)}(0)=-(2)^3$
$f^{(5)}(x)=-2^5\sin(2x)$	$f^{(5)}(0)=0$
$f^{(6)}(x)=-2^6\cos(2x)$	$f^{(6)}(0)=2^4$

$f^{(2k)}(0)=(-1)^k(2)^{2k}$

$$P_4(x)=1-\frac{2^2}{2!}x^2+\frac{2^4x^4}{4!}-\frac{2^6x^6}{6!}$$

$$=1-2x^2+\frac{4}{3}x^4$$

Mac Series: $\sum\frac{(-1)^k2^{2k}x^{2k}}{(2k)!}$

Radius: (Ratio Ext) $\lim_{k\to\infty}\left|\frac{(-1)^{k+1}2^{2k+2}x^{2k+2}}{(2k+2)!}\cdot\frac{(2k)!}{(-1)^k2^{2k}x^{2k}}\right|=\lim_{k\to\infty}\left|\frac{2^2x^2}{(2k+2)(2k+1)}\right|\overset{HP}{=}0<1$ $\boxed{R=\infty}$

$(-\infty,\infty)$

#3

$f(x) = (1+2x)^{-1}$ $\qquad f(0) = 1$

$f'(x) = -1 \cdot 2 \cdot (1+2x)^{-2}$ $\qquad f'(0) = -2$

$f''(x) = 2 \cdot 2^2 (1+2x)^{-3}$ $\qquad f''(0) = 2 \cdot 2^2$

$f'''(x) = -3! \cdot 2^3 (1+2x)^{-4}$ $\qquad f'''(0) = -3!\, 2^3$

$f^{(4)}(x) = 4! \cdot 2^4 (1+2x)^{-5}$ $\qquad f^{(4)}(0) = 4!\, 2^4$

$\Rightarrow f^{(k)}(0) = (-1)^k k!\, 2^k$

$$p_3(x) = 1 - \frac{2x}{1!} + \frac{8}{2!}x^2 - \frac{3!\, 2^3 x^3}{3!} = 1 - 2x + 4x - 8x^3 + 16x^4 \cdots$$

Mac Series: $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k k!\, 2^k x^k}{k!} = \sum (-2)^k x^k$

#4 $\cos x$ @ ctr $a = \pi$

$f(x) = \cos x$ $\qquad f(\pi) = -1$

$f'(x) = -\sin x$ $\qquad f'(\pi) = 0$

$f''(x) = -\cos x$ $\qquad f''(\pi) = 1$

$f'''(x) = \sin x$ $\qquad f'''(\pi) = 0$

$f^{(4)}(x) = \cos x$ $\qquad f^{(4)}(\pi) = -1$

$f^{(5)}(x) = -\sin x$ $\qquad f^{(5)}(\pi) = 0$

$f^{(6)}(x) = -\cos x$ $\qquad f^{(6)}(\pi) = 1$

$f^{(k)}(\pi) = (-1)^{2k+1}$

$(x-\pi)$

$$p_6(x) = -1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!} + \frac{(x-\pi)^6}{6!}$$

starts w/ -1

Taylor Series: $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2k)!} x^{2k}$

#5 $f(x) = 1/x$ @ $x = 2$

$f(x) = 1/x = x^{-1}$ $\qquad f(2) = 1/2$

$f'(x) = -x^{-2}$ $\qquad f'(2) = -1/2^2$

$f''(x) = 2x^{-3}$ $\qquad f''(2) = 2!\, \frac{1}{2^3}$

$f'''(x) = -3!\, x^{-4}$ $\qquad f'''(2) = -3!\, 1/2^4$

$f^{(4)}(x) = 4!\, x^{-5}$ $\qquad f^{(4)}(2) = 4!\, 1/2^5$

$$f^{(k)}(2) = \frac{(-1)^k k!}{2^{k+1}}$$

$$p_3(x) = 1/2 - \frac{(x-2)}{4} + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$$

Taylor Series $= \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k k!}{2^{k+1}\, k!}(x-2)^k = \sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^k}{2^{k+1}}$